OEUVRE: OnlinE Unbiased Variance-Reduced loss Estimation

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Problem: Online Loss Estimation

- Consider sequential data $\{z_t\}_{t \geq 1}$, where $z_t = (x_t, y_t) \sim \mathscr{D}$
- Learning algorithm $\mathscr{A}$ trained on data, with function $f_t : \mathscr{X} \to \mathscr{Y}$ learned using first $(t-1)$ samples
- **Goal**: Obtain accurate estimate of expected loss for currently learned function

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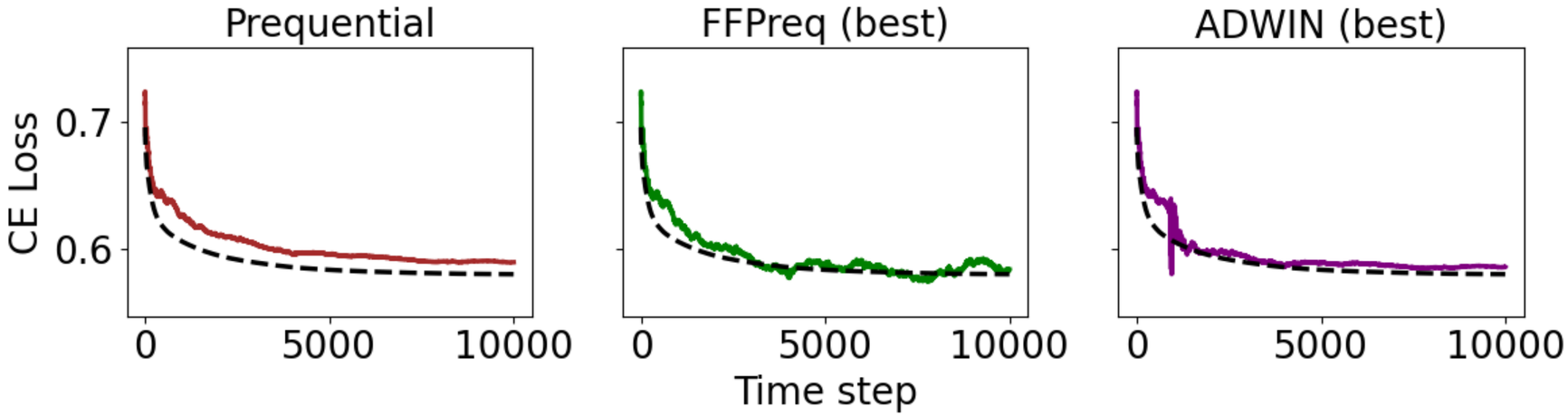
Applications
Online model monitoring
Early stopping
Online model selection

Desirable Estimator Properties

We want an estimator with

- quick and efficient updates,
- provably strong convergence properties, and
- use of properties of the learning algorithm $\mathscr{A}$



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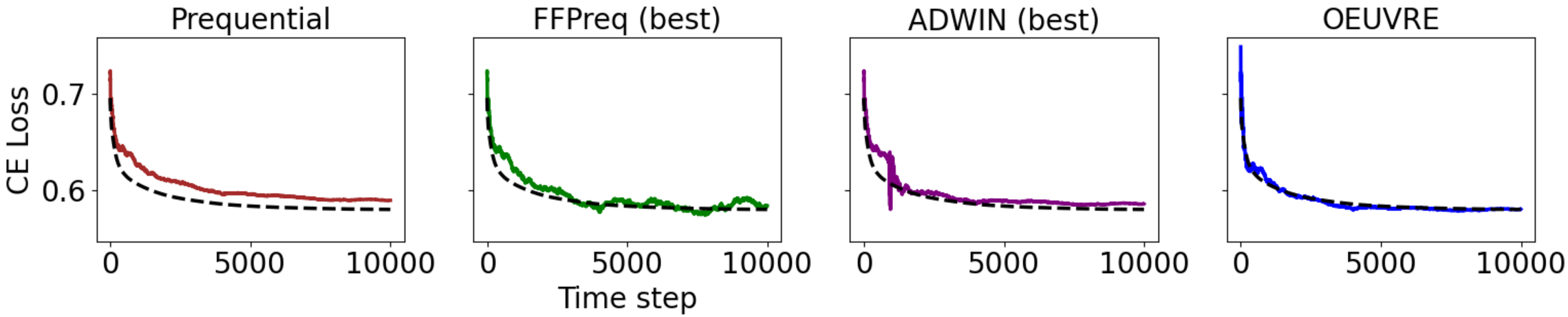

Enter *OEUVRE*!

OEUVRE Estimator

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Weight term.
Set γ_t according to learning algorithm!

Variance reduction term

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Algorithmic paradigm	Stability bound
Follow the leader (FTL)	$O(1/t)$
Follow the regularized leader (FTRL)	$O(1/\sqrt{t})$
Regularized dual averaging (RDA)	$O(1/\sqrt{t})$
Online mirror descent (OMD)	$O(\eta_t)$

Theoretical Properties

OEUVRE

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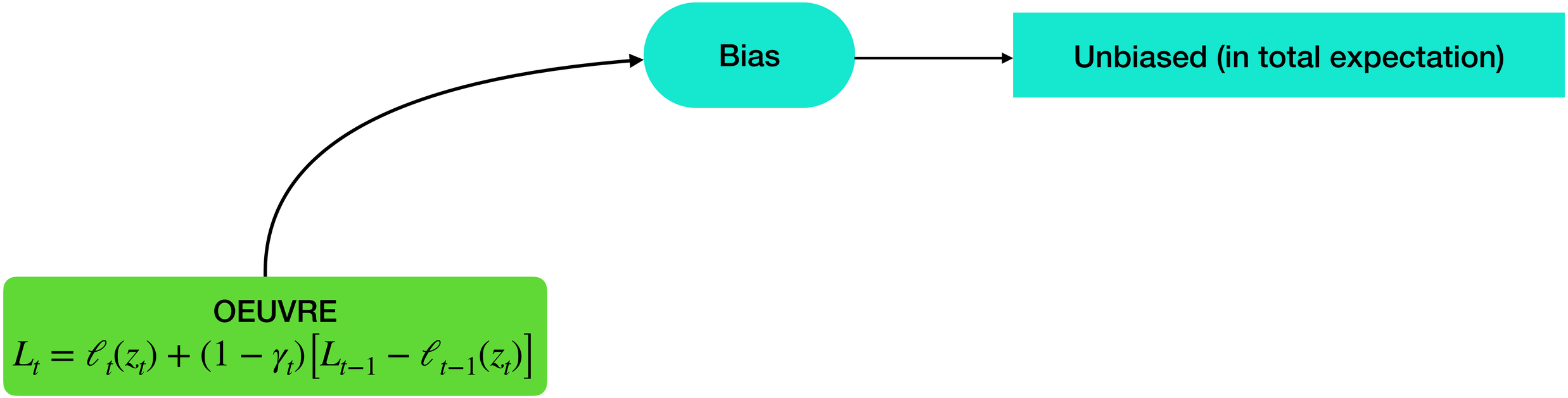

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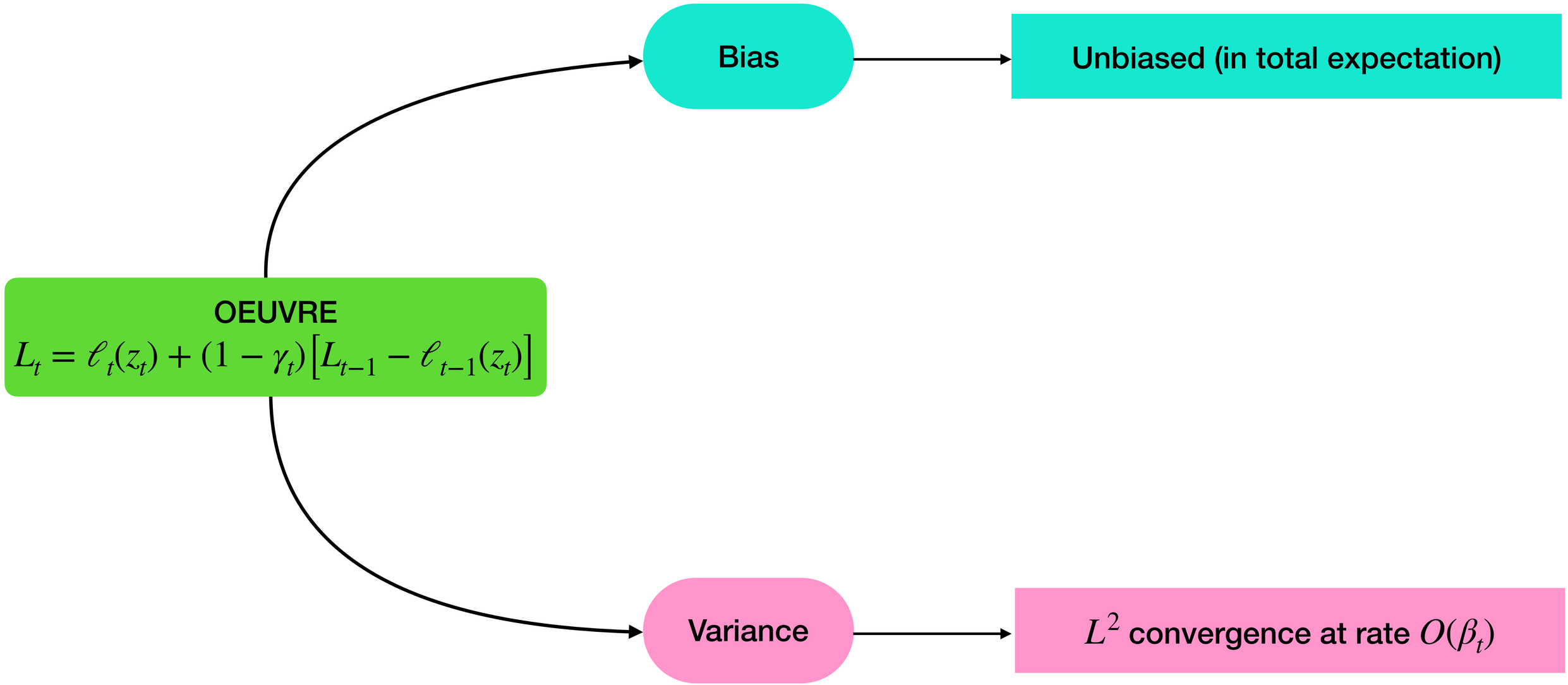

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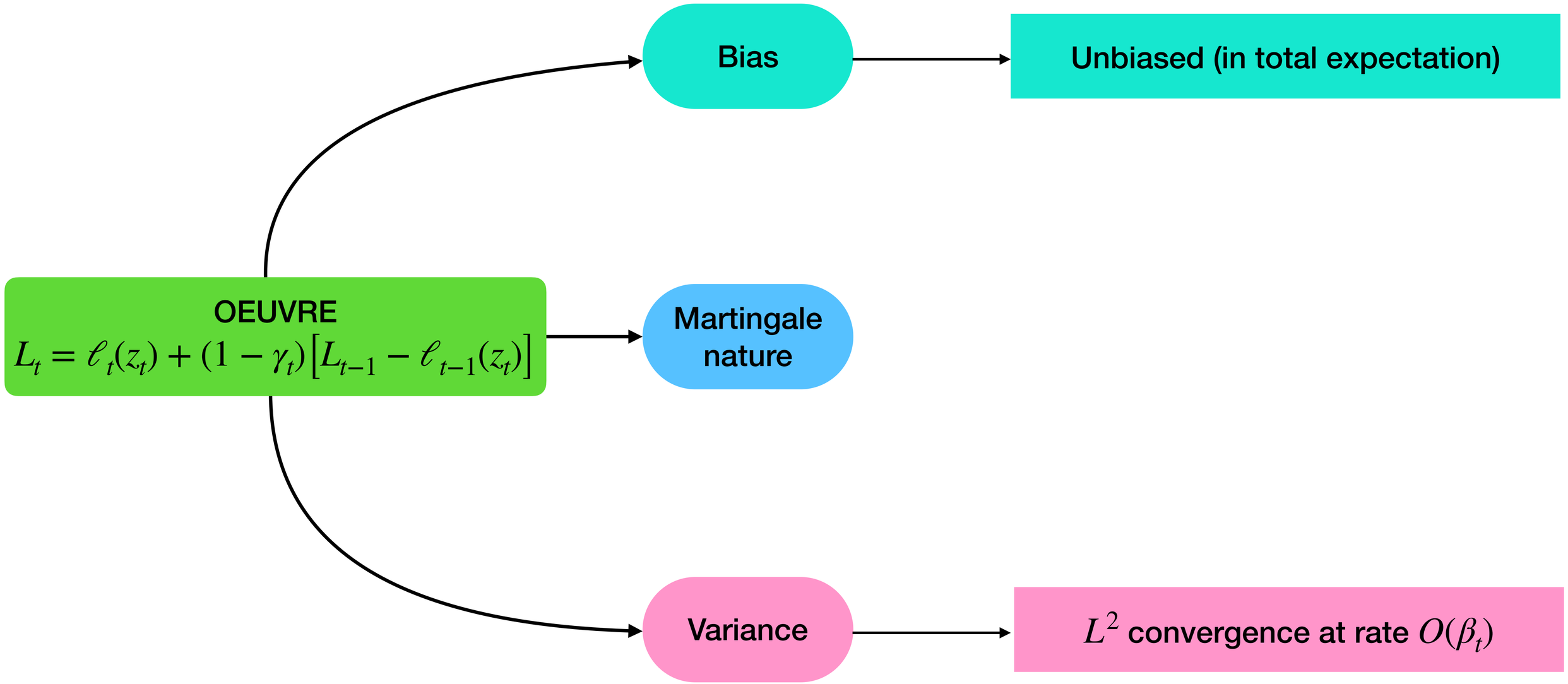

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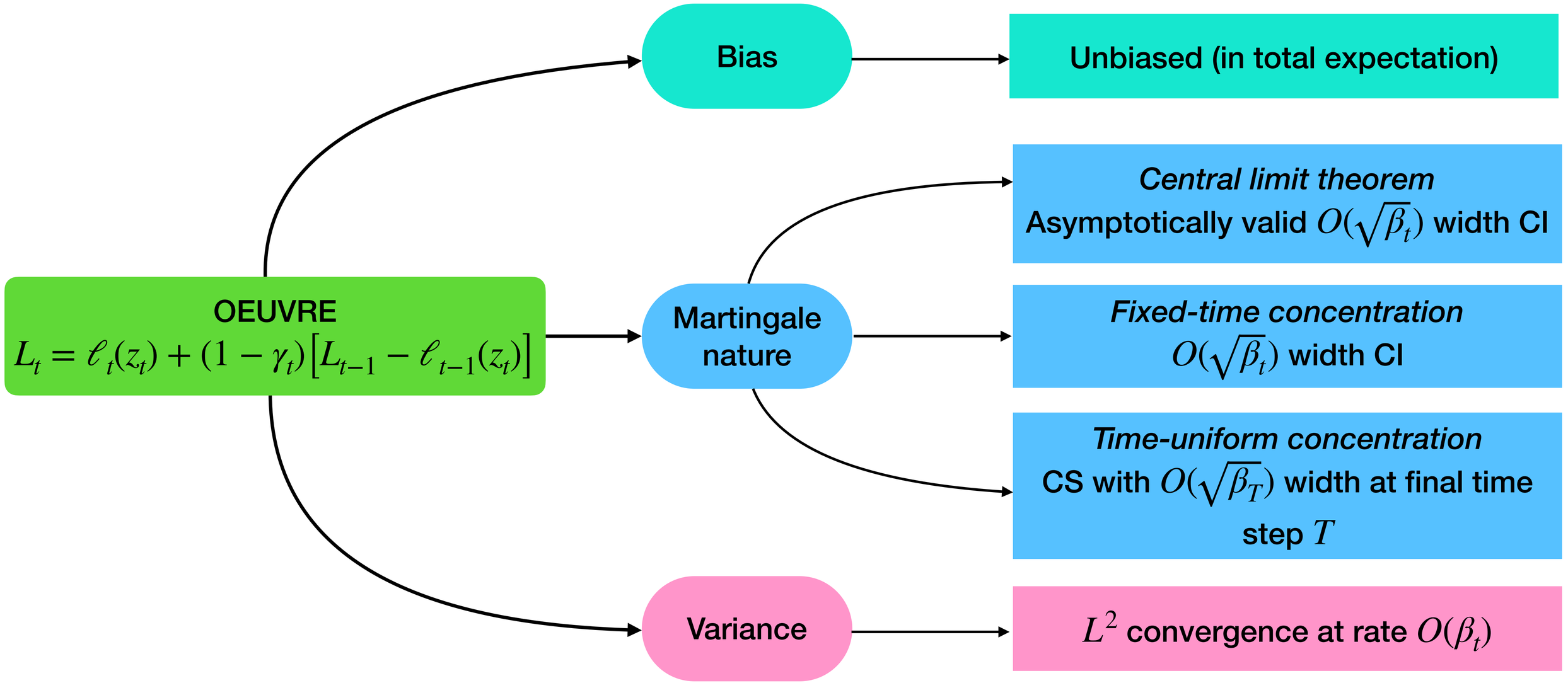

Summary

- OEUVRE: extra evaluation at each time step for variance reduction
- Leverage algorithmic stability properties to design plug-and-play estimator
- Strong statistical properties (convergence, confidence intervals)
- Hyperparam-free OEUVRE: Adaptively estimate constants in stability rate
- Experiments:
    1. Linear regression
    2. Prediction with expert advice
    3. Simple neural networks

Check out our paper!
Poster #173
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