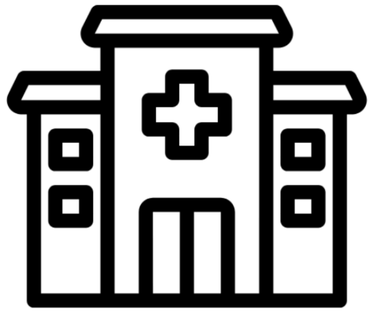
EventFlow: Forecasting Temporal Point Processes with Flow Matching

Gavin Kerrigan[13], Kai Nelson[23], Padhraic Smyth[3]

[1]University of Oxford [2]UC Berkeley [3]UC Irvine

Event data occurring at irregular times



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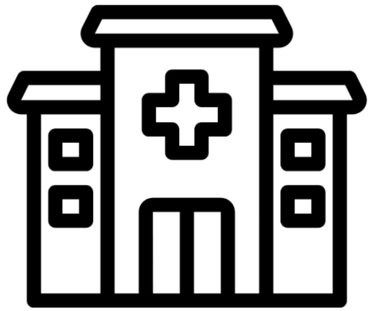

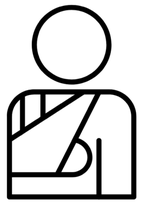

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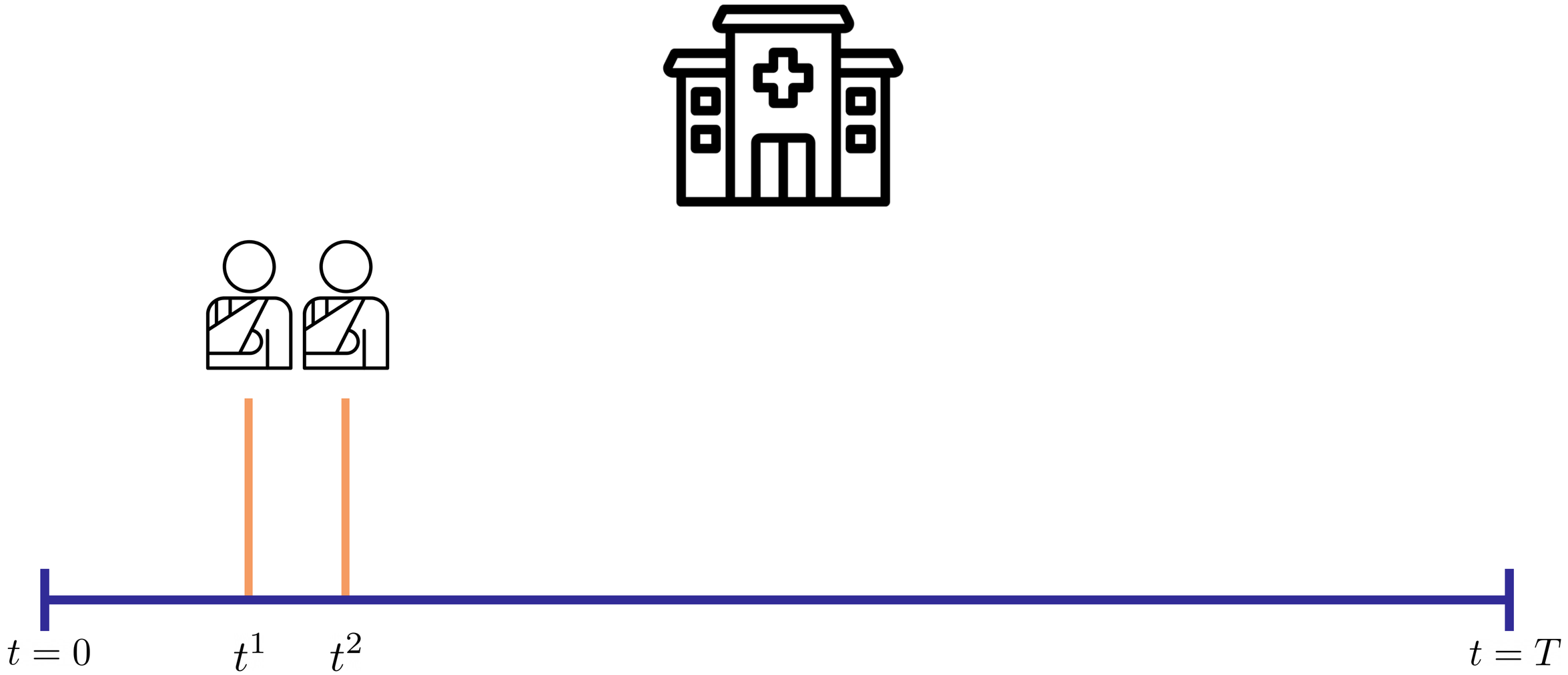

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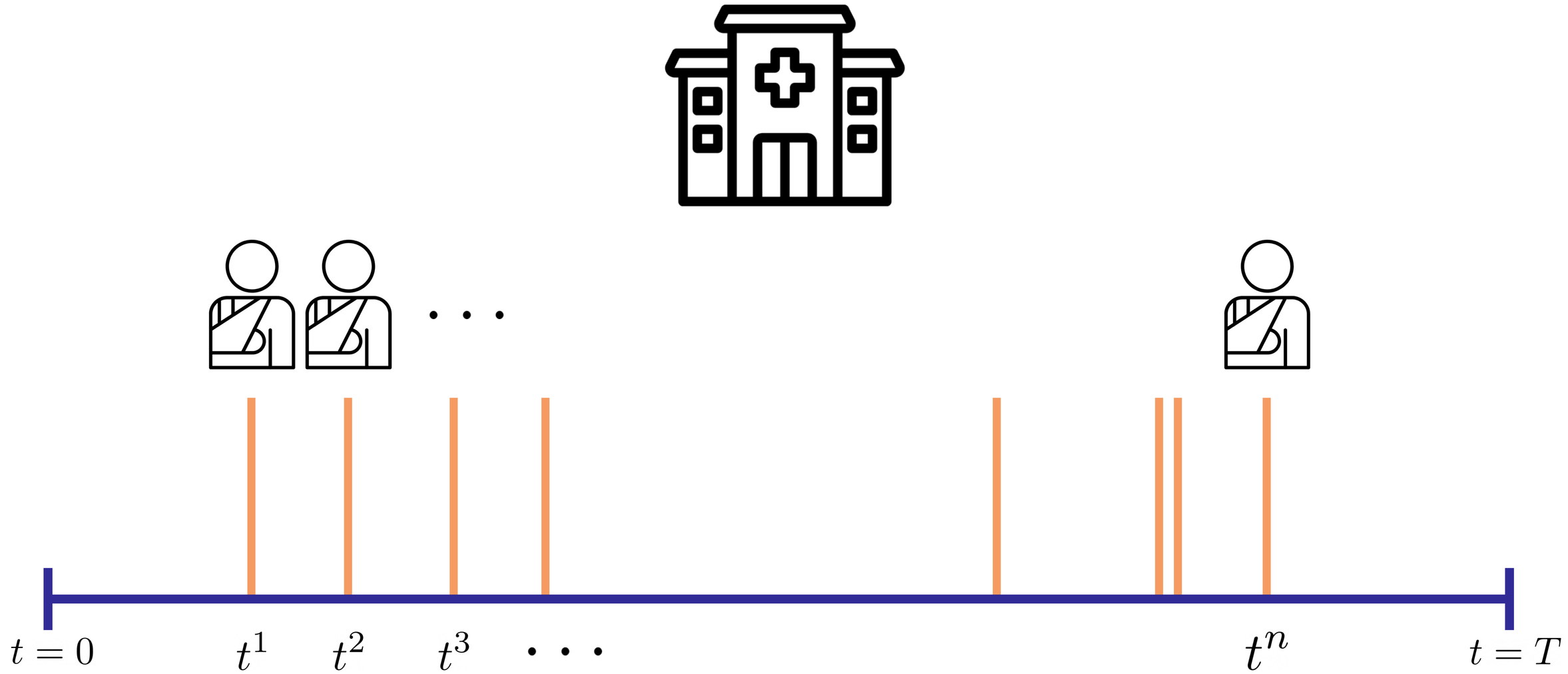
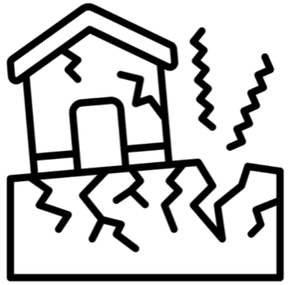
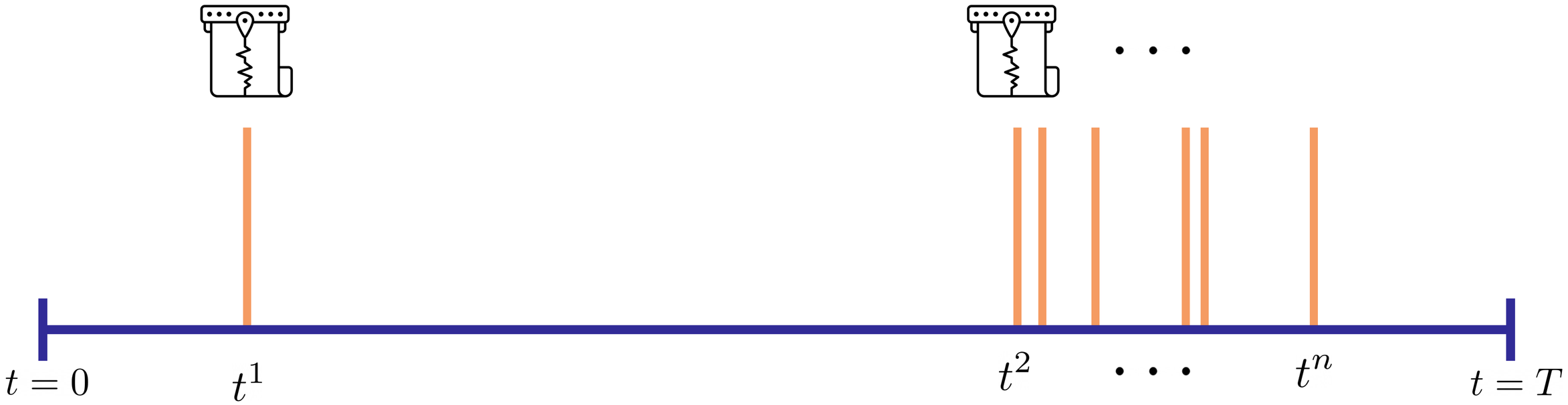


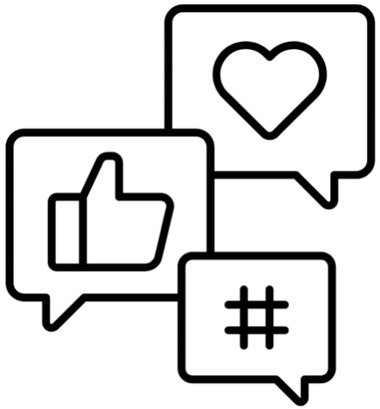
Event data occurring at irregular times

Often involve complex, varied patterns





Social Media



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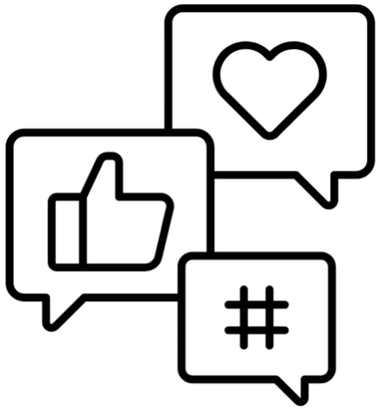
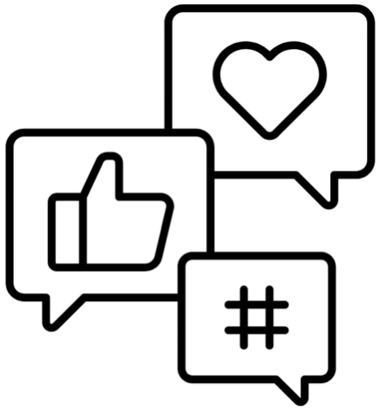


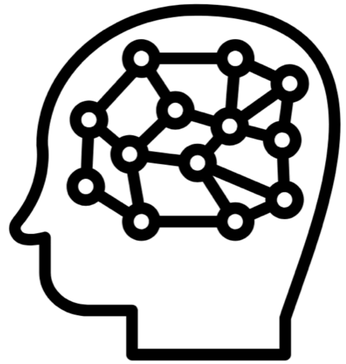
Consumer Behaviour

Social Media



Consumer Behaviour

Brain Activity

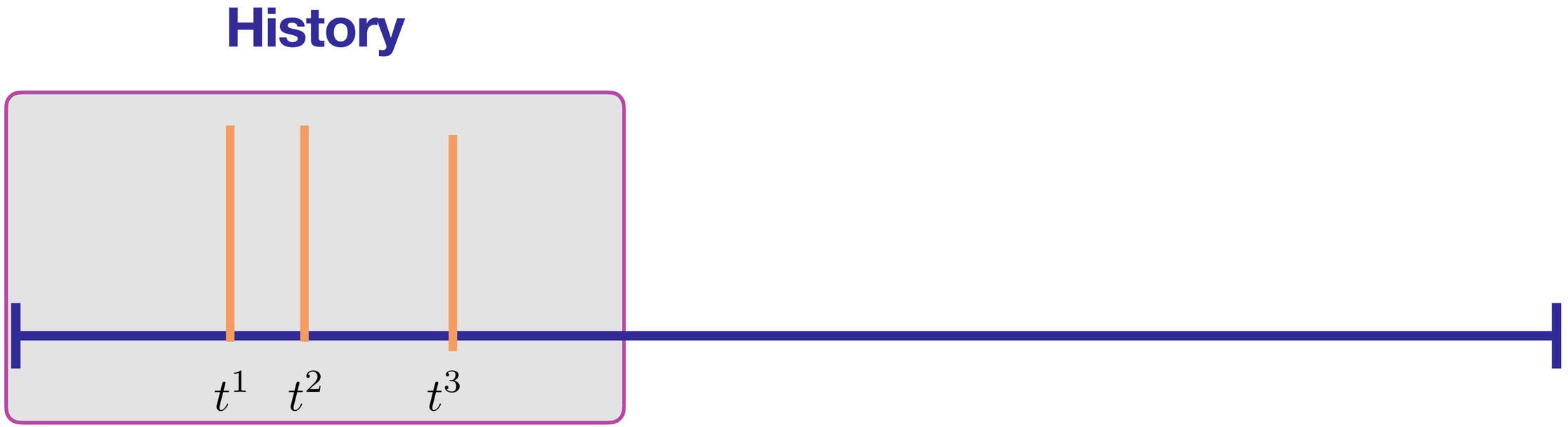


Key Challenge: Forecasting

When will future events occur?

Key Challenge: Forecasting

Standard approaches are largely autoregressive



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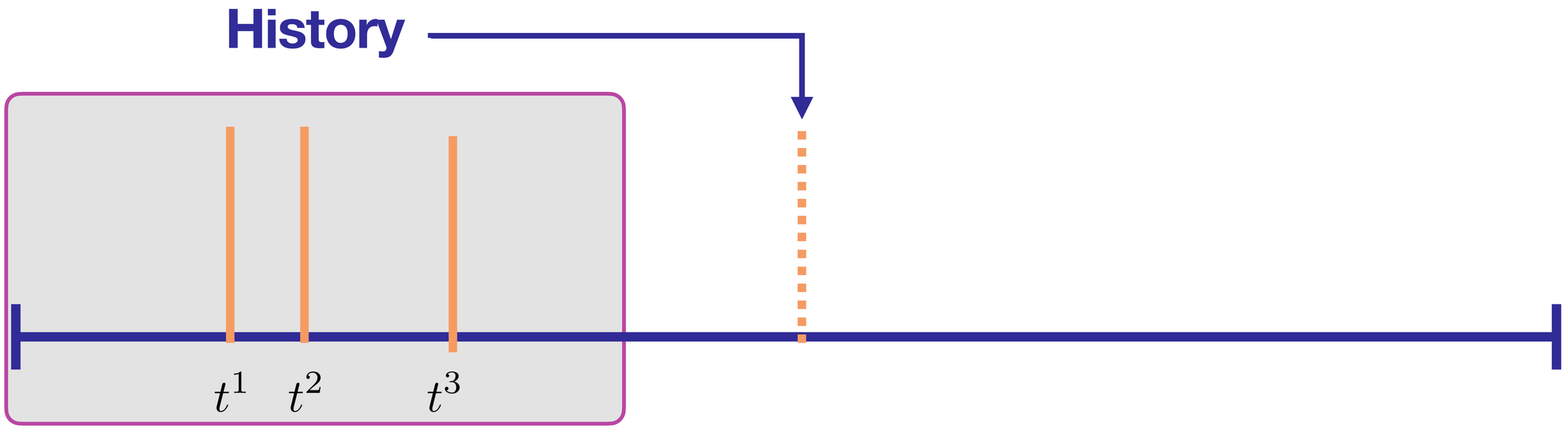

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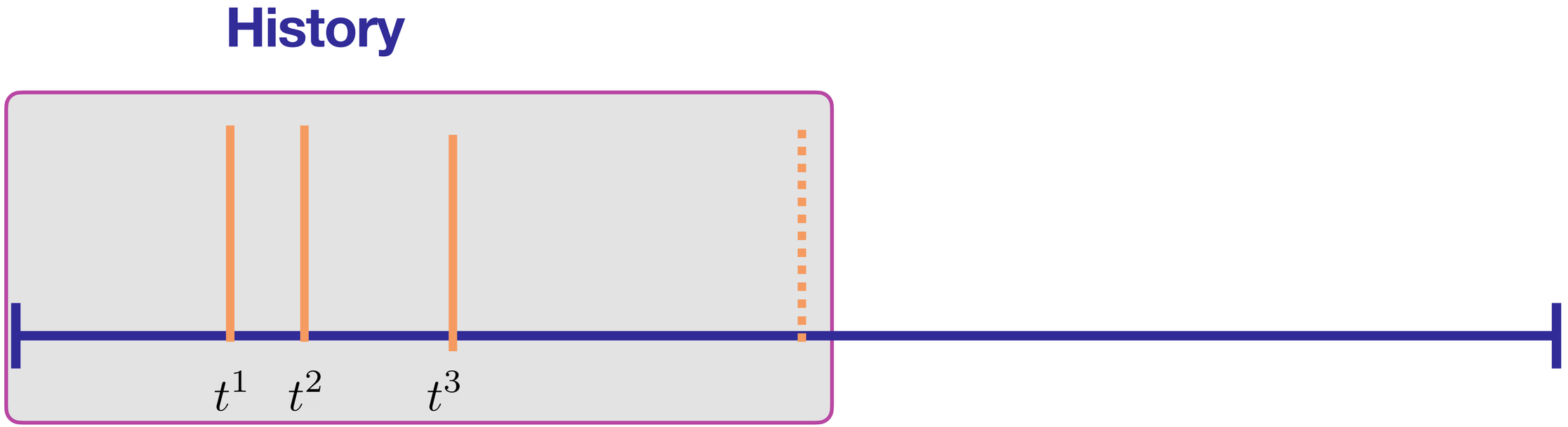

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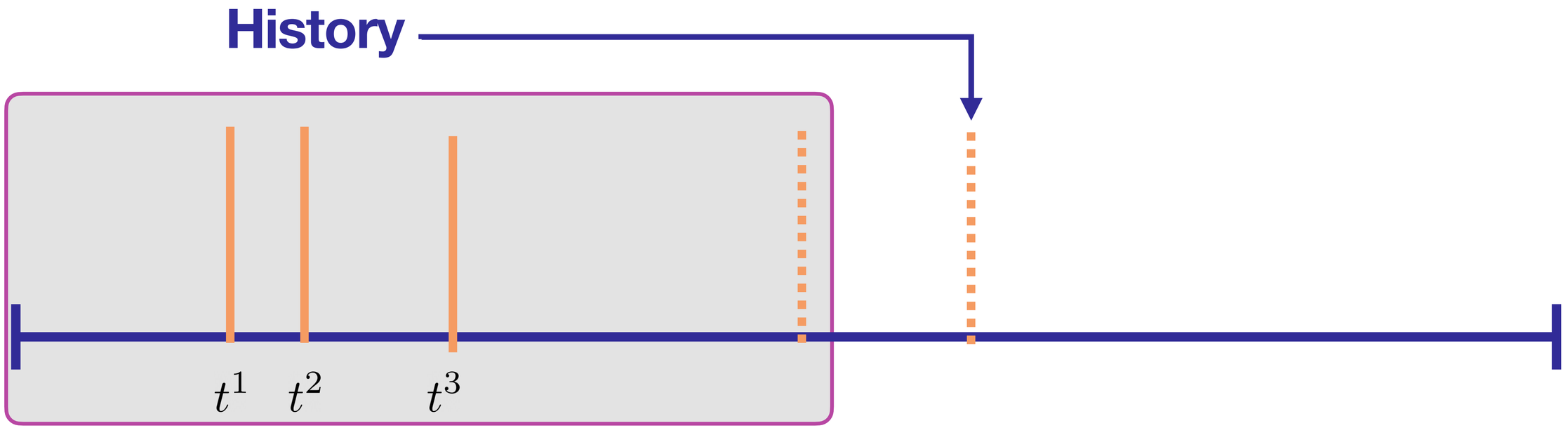

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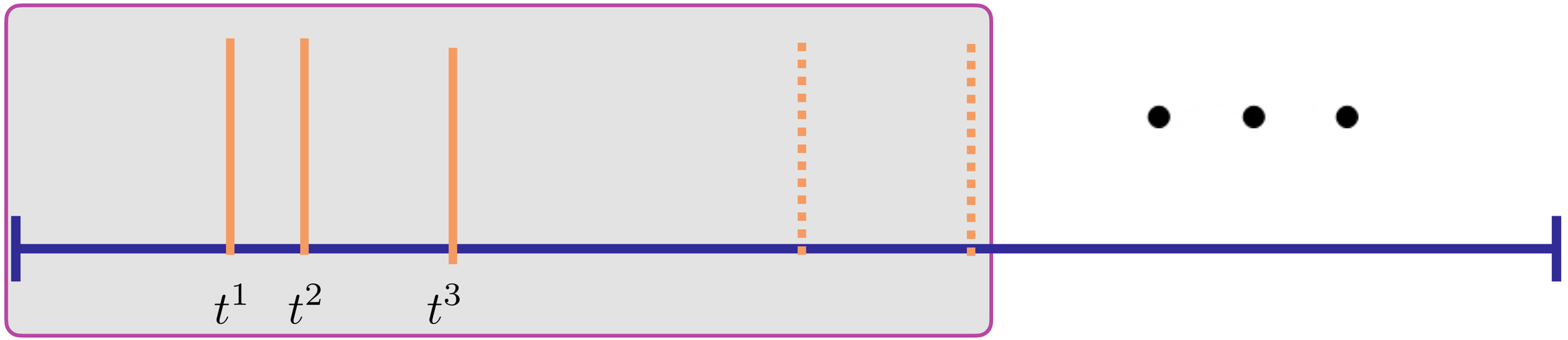
History



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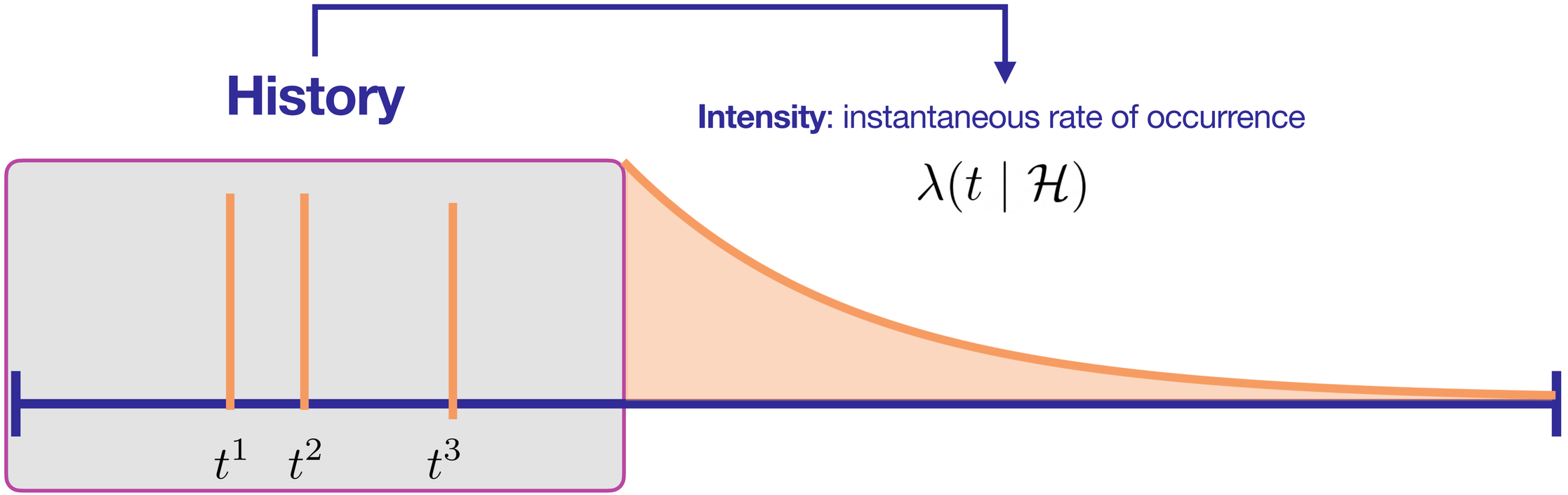

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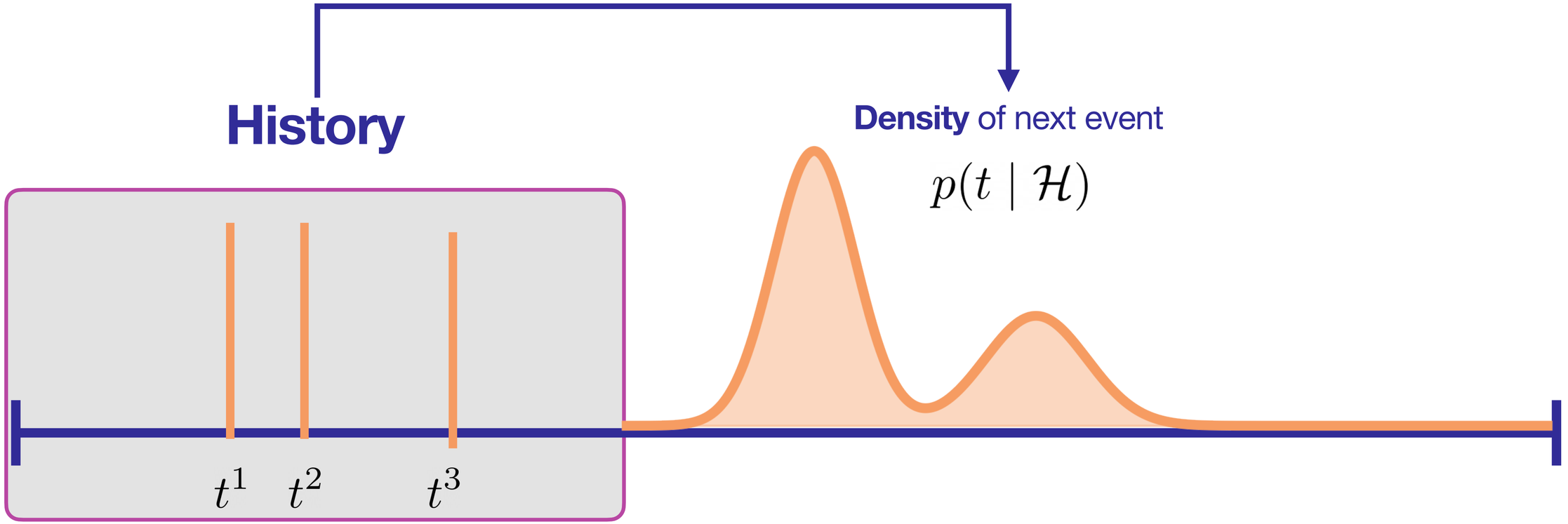
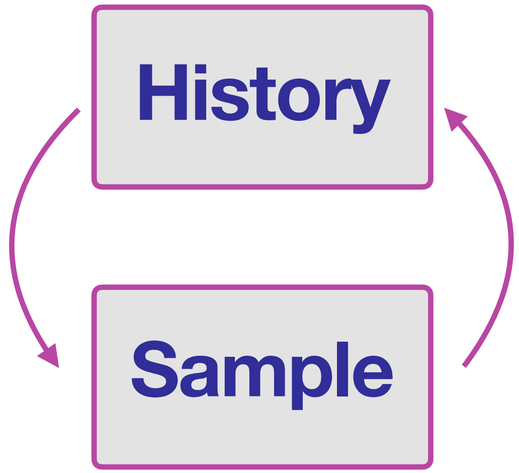


Limitations of Autoregressive Models

Reliance on simple or hand-crafted forms for the intensity/density

- Requirement: able to evaluate likelihood to train
- More expressive choices quickly become expensive



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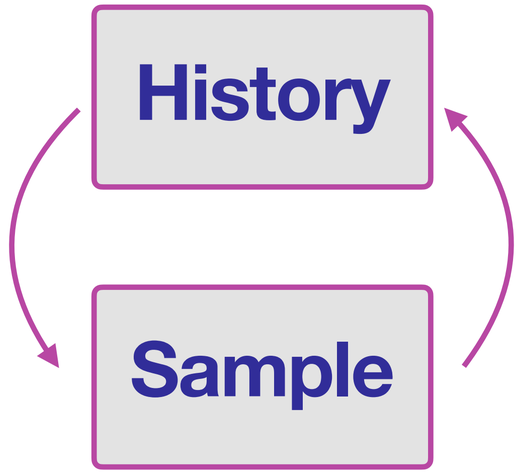

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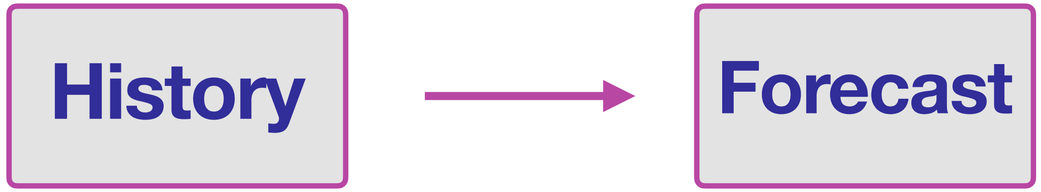
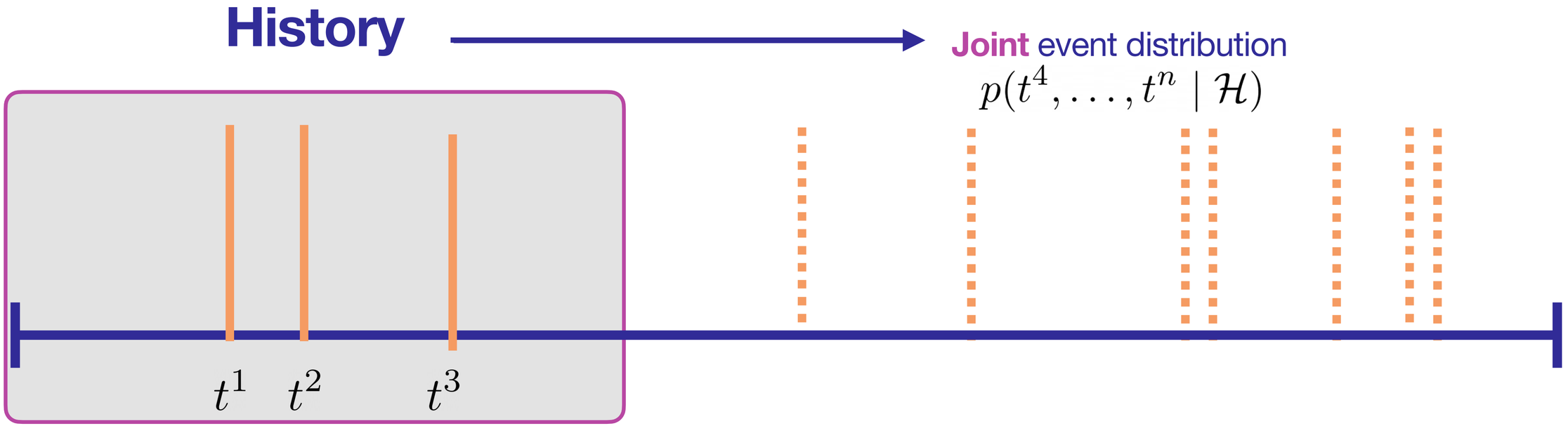
EventFlow: directly predict the full forecast



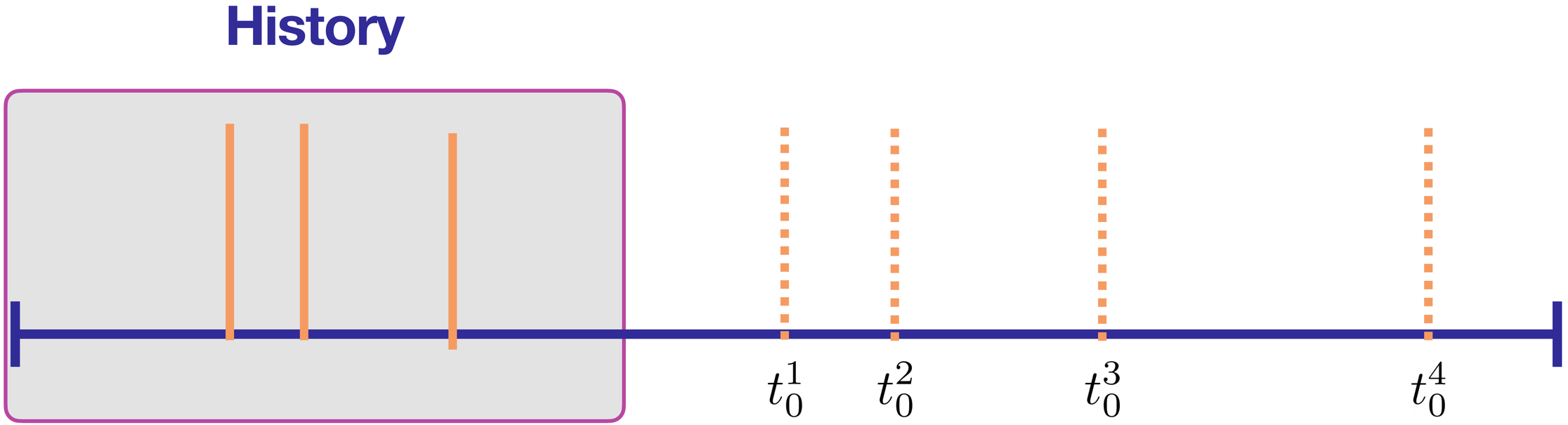
EventFlow: Forecasting Event Sequences



EventFlow: Forecasting Event Sequences

[1] Initialise a forecast randomly

$$t_0^i \sim q(\cdot) \qquad i = 1, 2, \ldots, n$$



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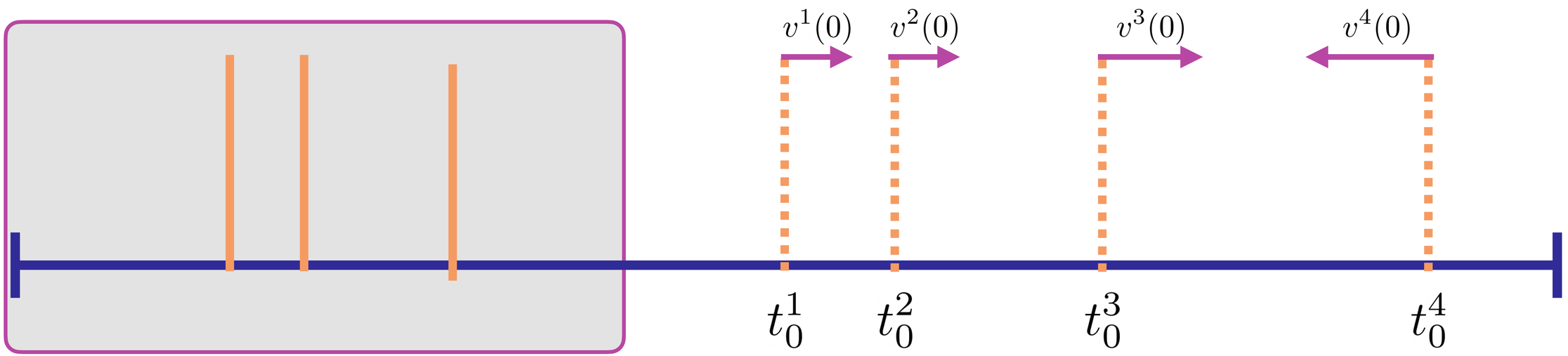

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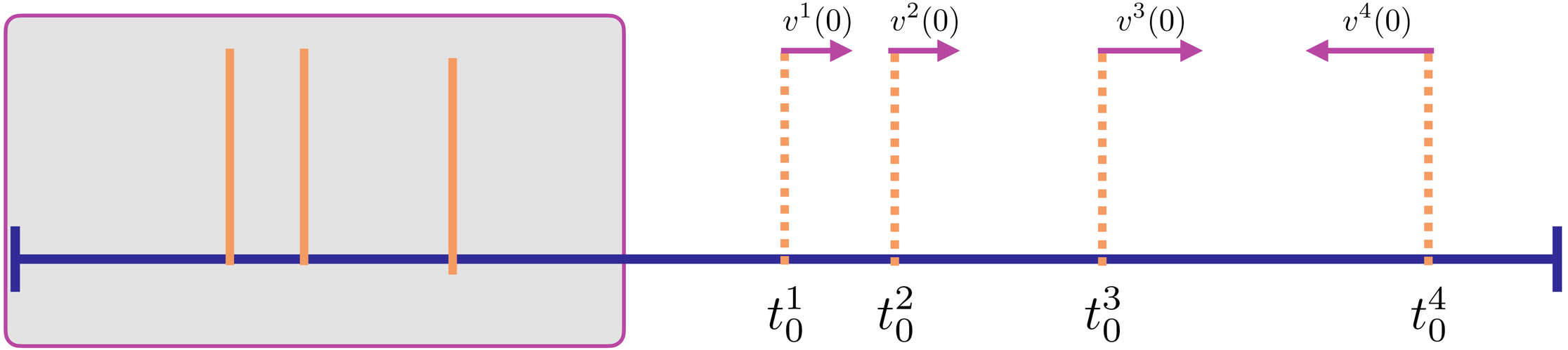
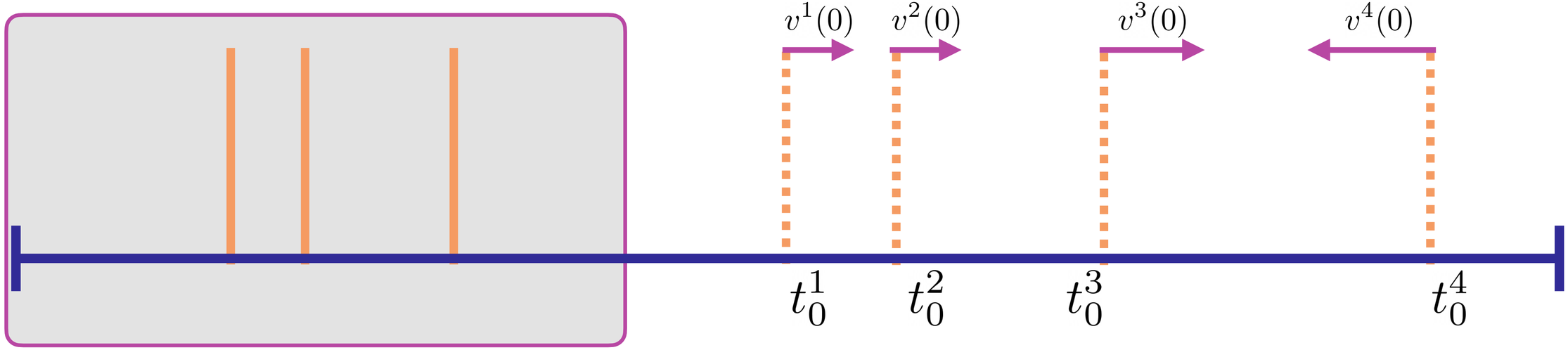
[3] Flow each event according to its velocity $dt_s^i = v^i(s, \mathbf{t}_s)ds$

History



History

s = 0



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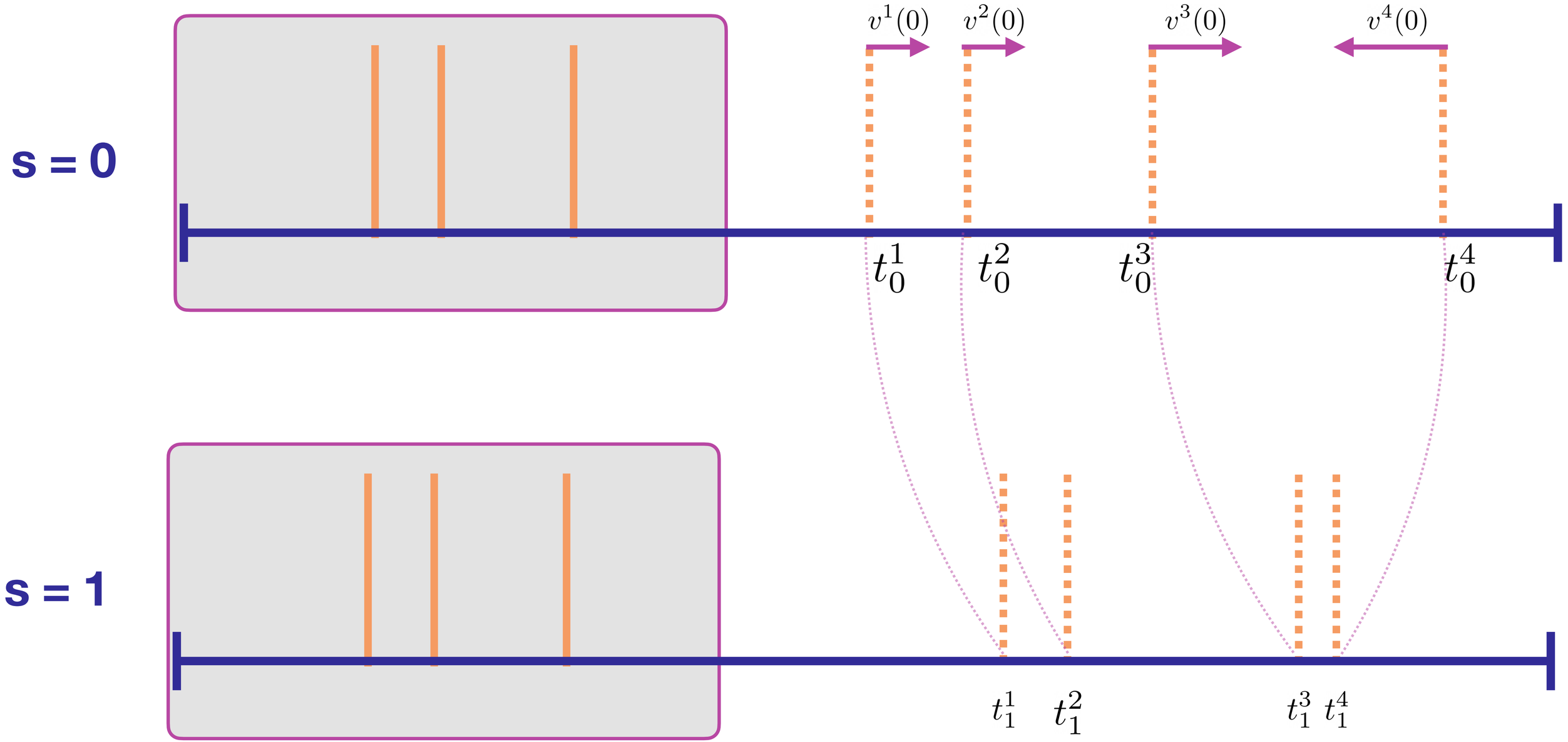
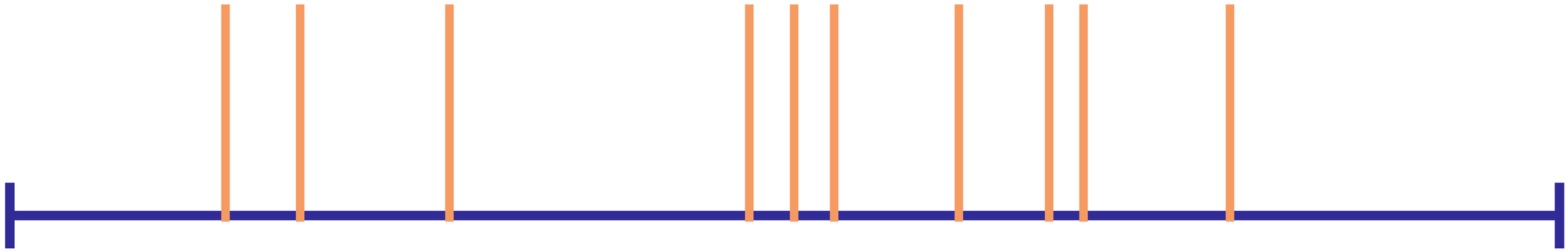


Training EventFlow



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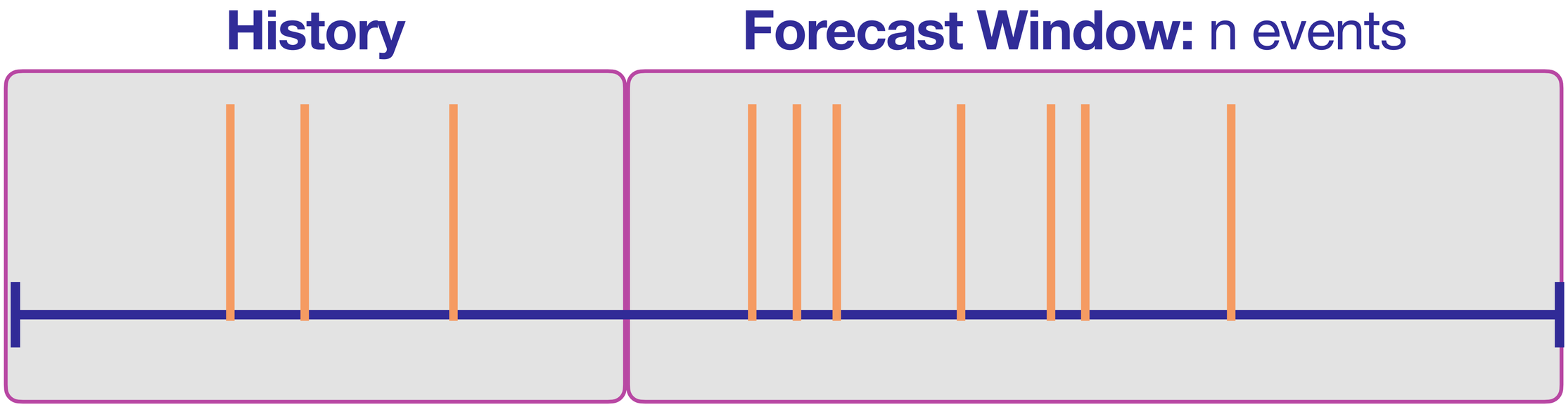

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Inspired by Flow Matching [Lipman et al. 2023; Tong et al. 2024]

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[3] Sample flow time uniformly and interpolate $t_s^i = (1-s)t_0^i + st_1^i$

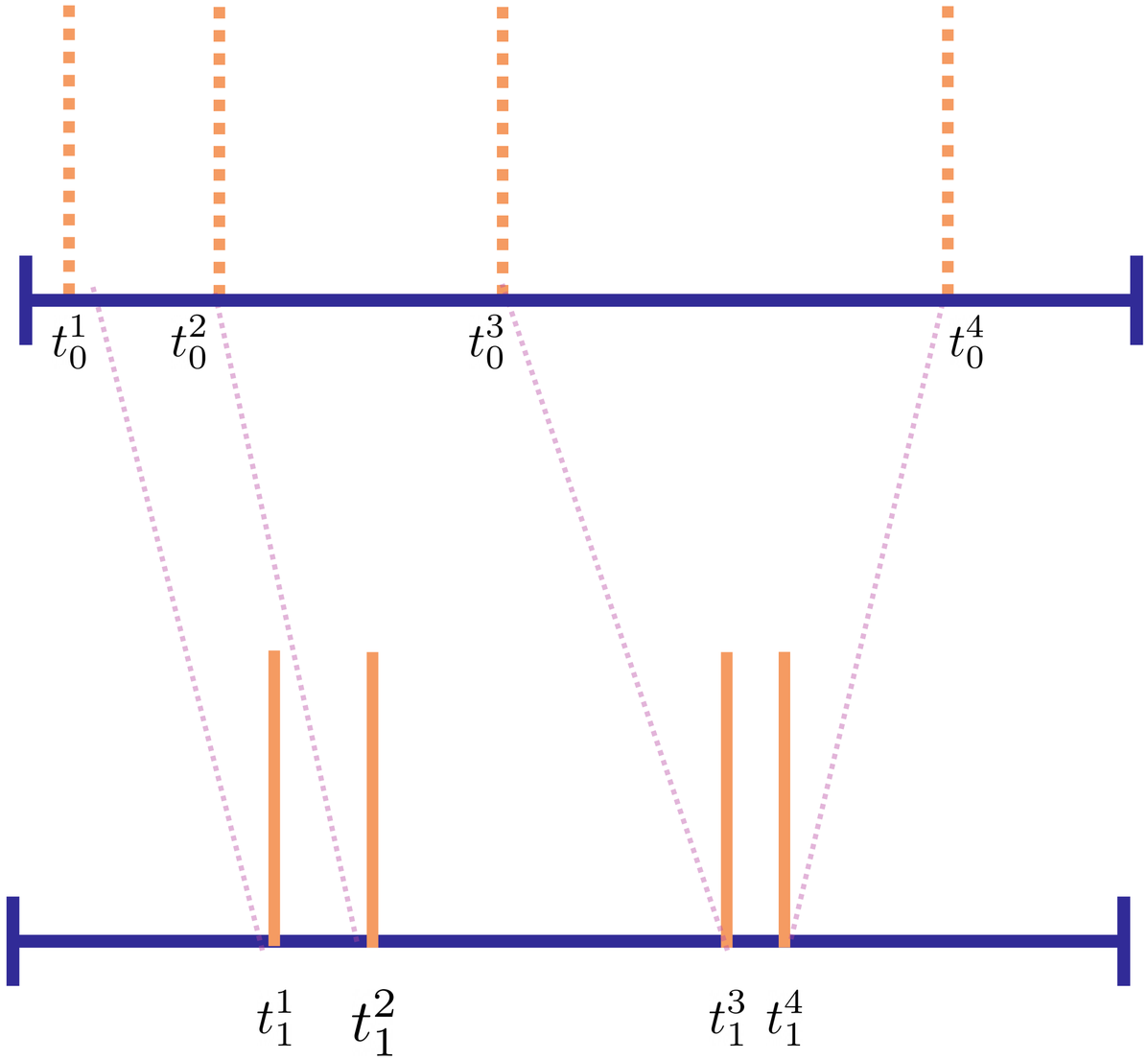
[4] Use noisy version to predict straight-line velocity

$$\sum_i |v_\theta^i(s, \mathbf{t}_s + \epsilon \mid \mathcal{H}) - (t_1^i - t_0^i)|^2 \qquad \epsilon \sim \mathcal{N}(0, \sigma^2 I)$$

Why does this work?

Interpolants define a path of sequences

$$\mathbf{t}_s = (1-s)\mathbf{t}_0 + s\mathbf{t}_1$$



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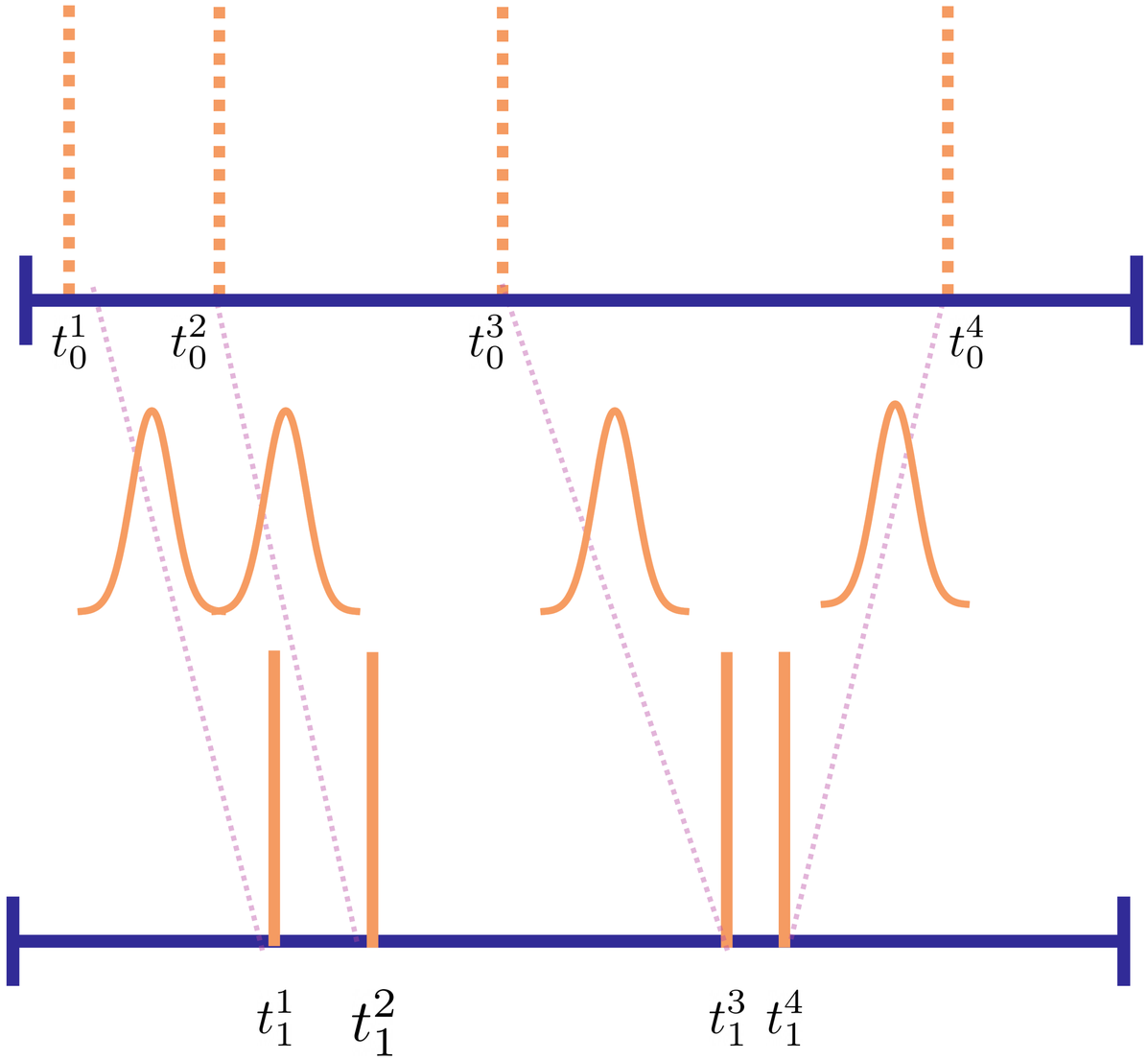

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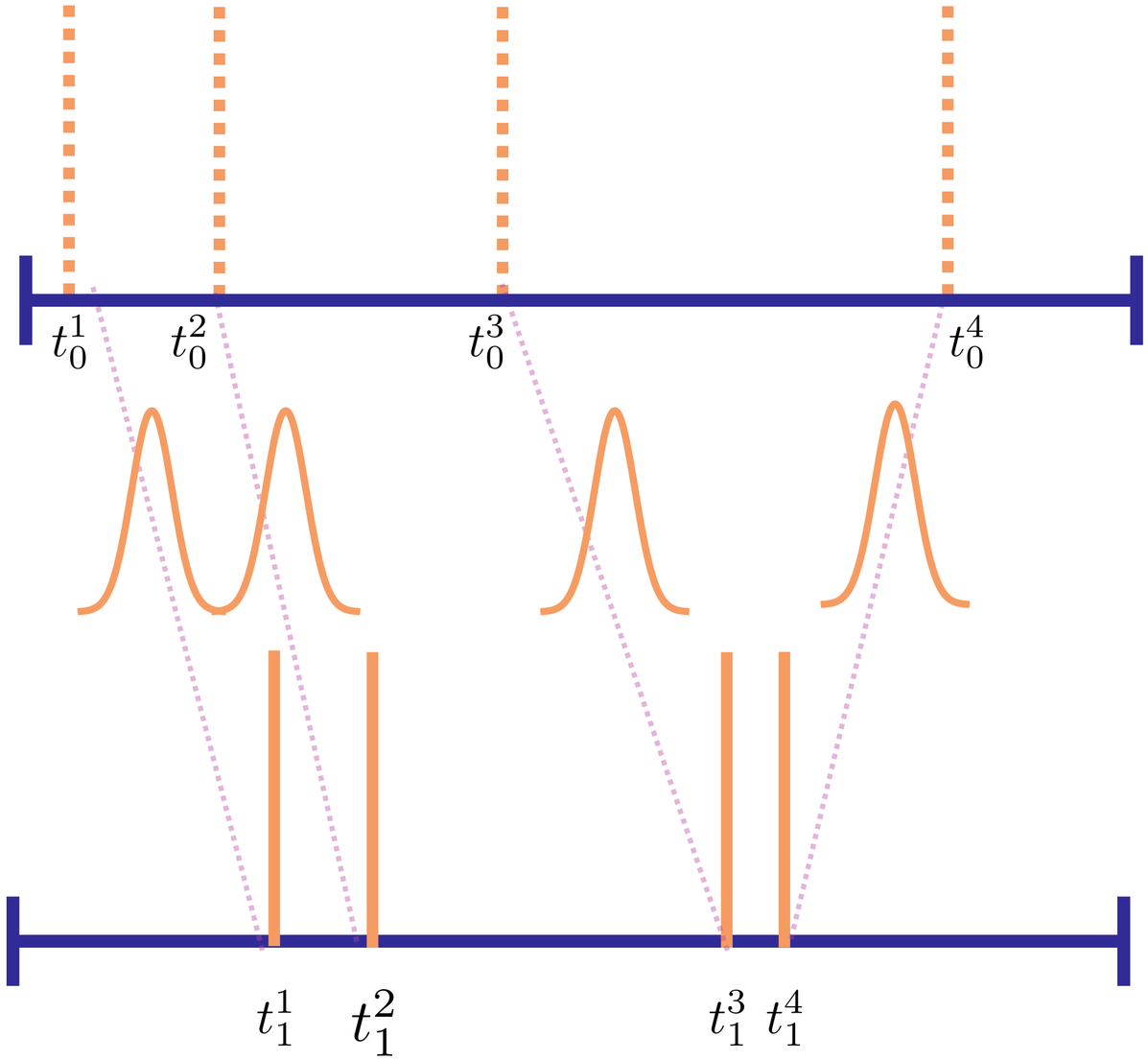
Marginalising gives a path of distributions
Random sequences —> Data Sequences

$$p_s(\cdot) = \int p_s(\cdot \mid \mathbf{t}_0, \mathbf{t}_1) d\pi(\mathbf{t}_0, \mathbf{t}_1)$$



Why does this work?

Marginal path of distributions
Interpolates random and data distributions

$$p_s(\cdot) = \int p_s(\cdot \mid \mathbf{t}_0, \mathbf{t}_1) d\pi(\mathbf{t}_0, \mathbf{t}_1)$$

Generated by the intractable vector field

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Regressing on the marginal velocity is equivalent to regressing on the conditionals:

$$\mathbb{E}|v_\theta(s, \mathbf{t}_s + \epsilon) - (\mathbf{t}_1 - \mathbf{t}_0)|^2$$

EventFlow fixes the number of events throughout generation

How do we handle variable-length forecasts?

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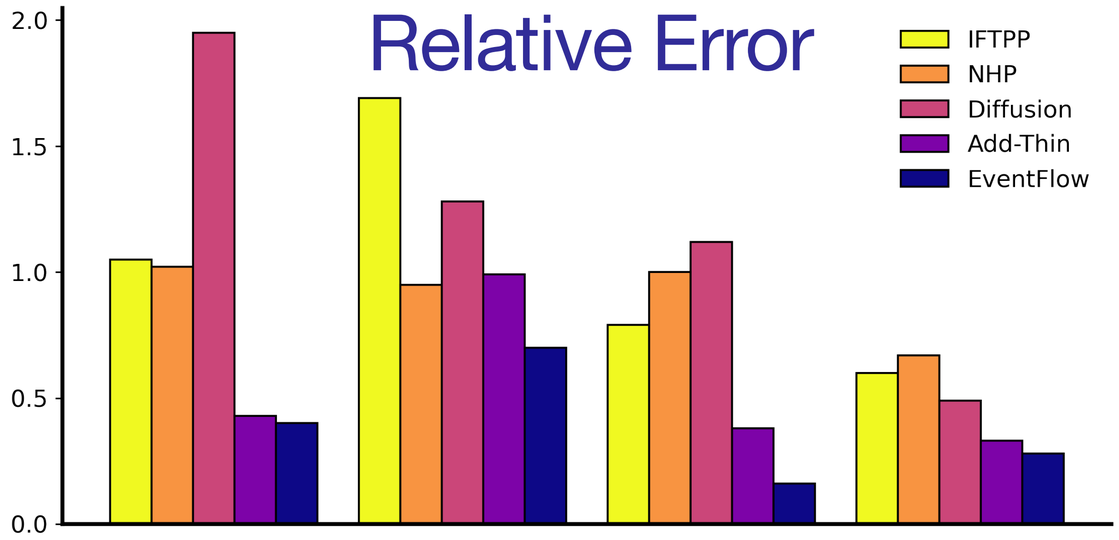

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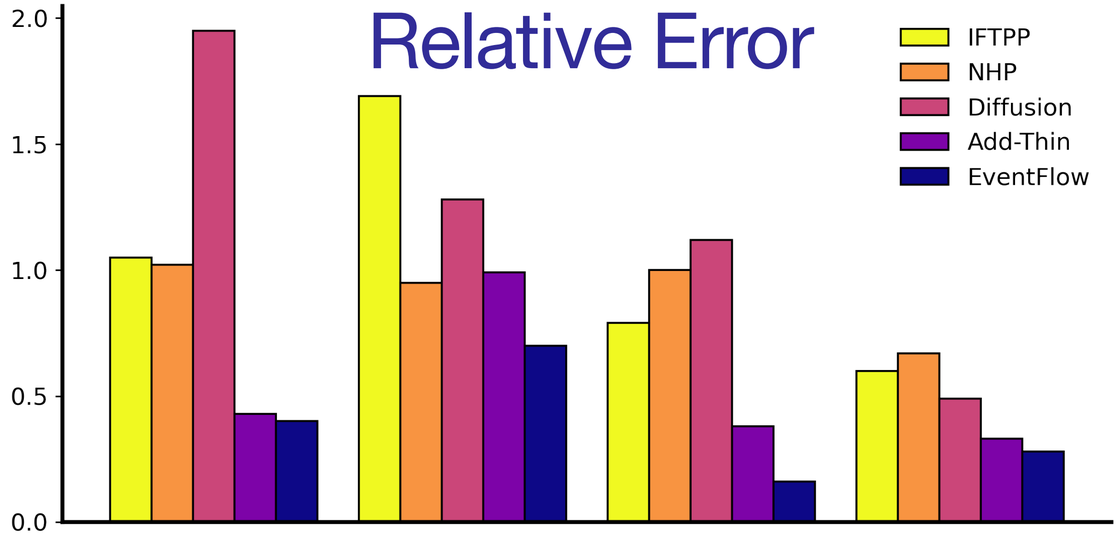
- Autoregressive learn this implicitly; we directly train for it
- Gives explicit control
  - What would the forecast be if n was large/small?



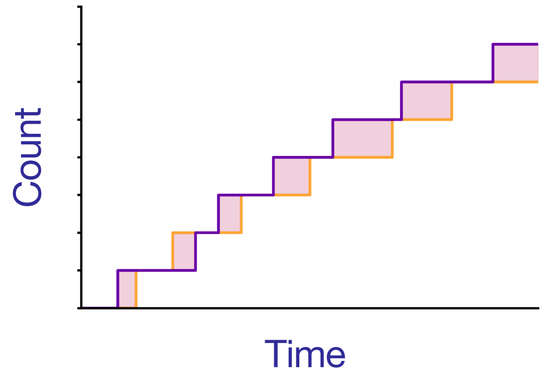
Experiments: Forecasting Real-World Event Sequences

Distance between sequences of **different lengths**:

[Xiao et al. 2017]

$$\mathbf{t} = \left(t^1, t^2, \ldots, t^n\right) \quad \hat{\mathbf{t}} = \left(\hat{t}^1, \hat{t}^2, \ldots, \hat{t}^m\right) \quad m > n$$

$$d(\mathbf{t}, \hat{\mathbf{t}}) = \sum_{k=1}^{n} \left|t^k - \hat{t}^k\right| + \sum_{k=n+1}^{m} \left(T - \hat{t}^k\right)$$



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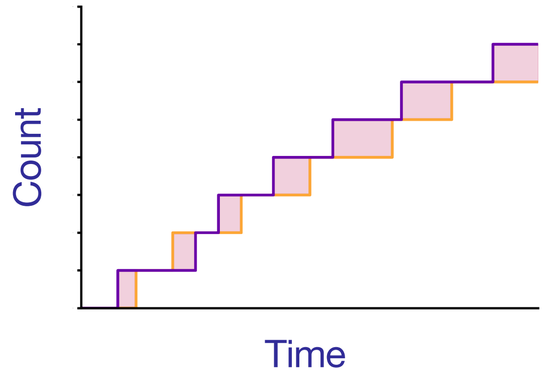

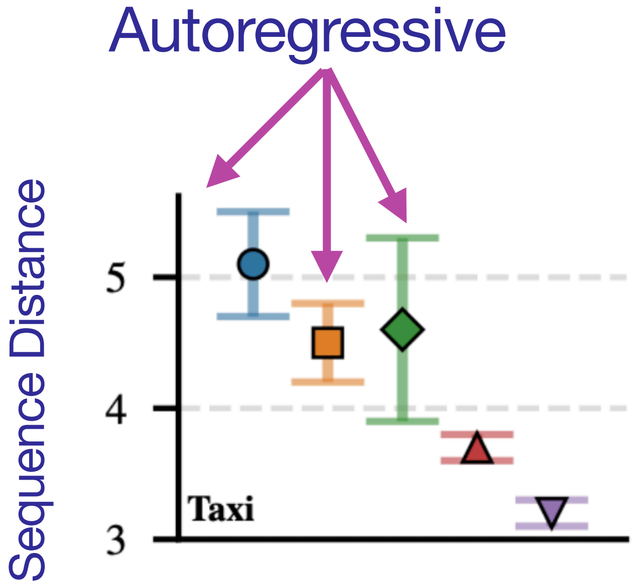

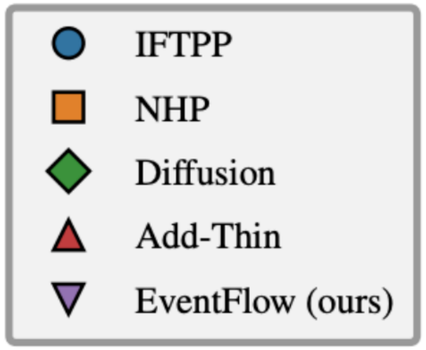

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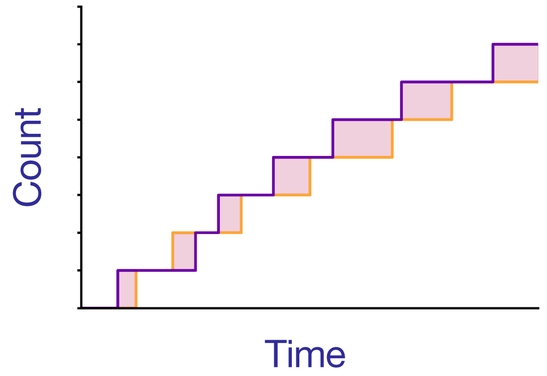

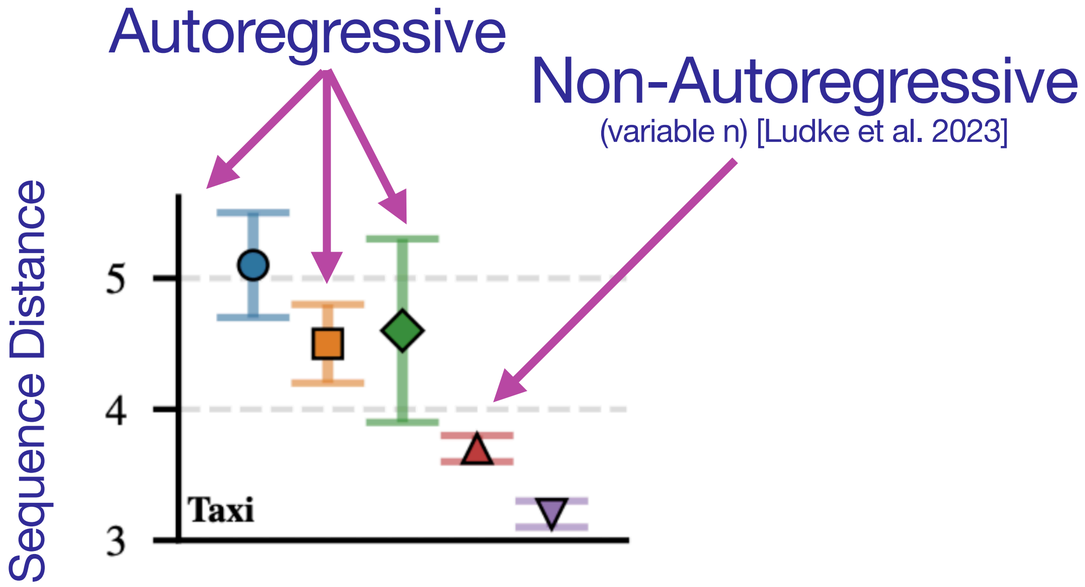

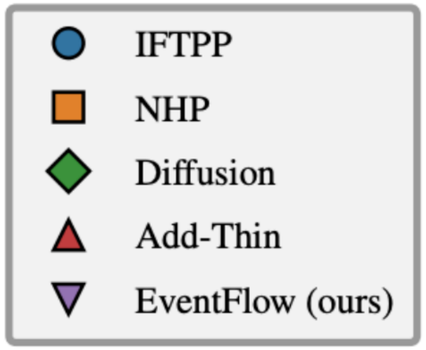

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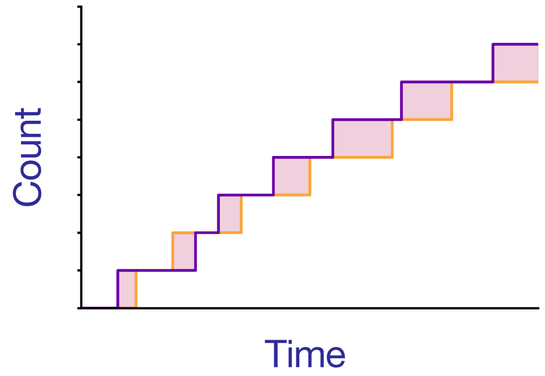
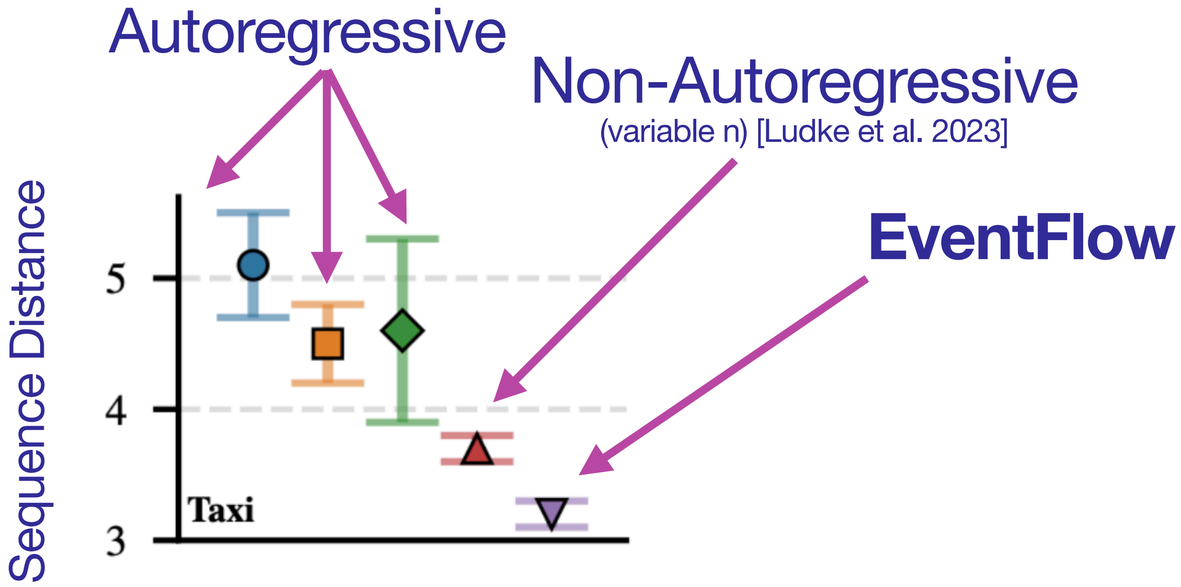
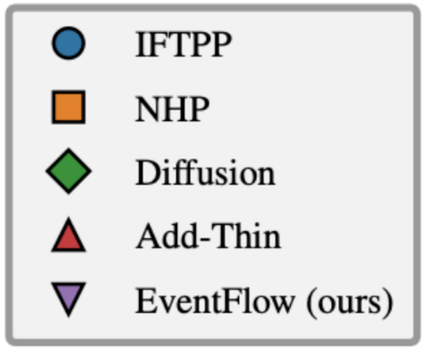
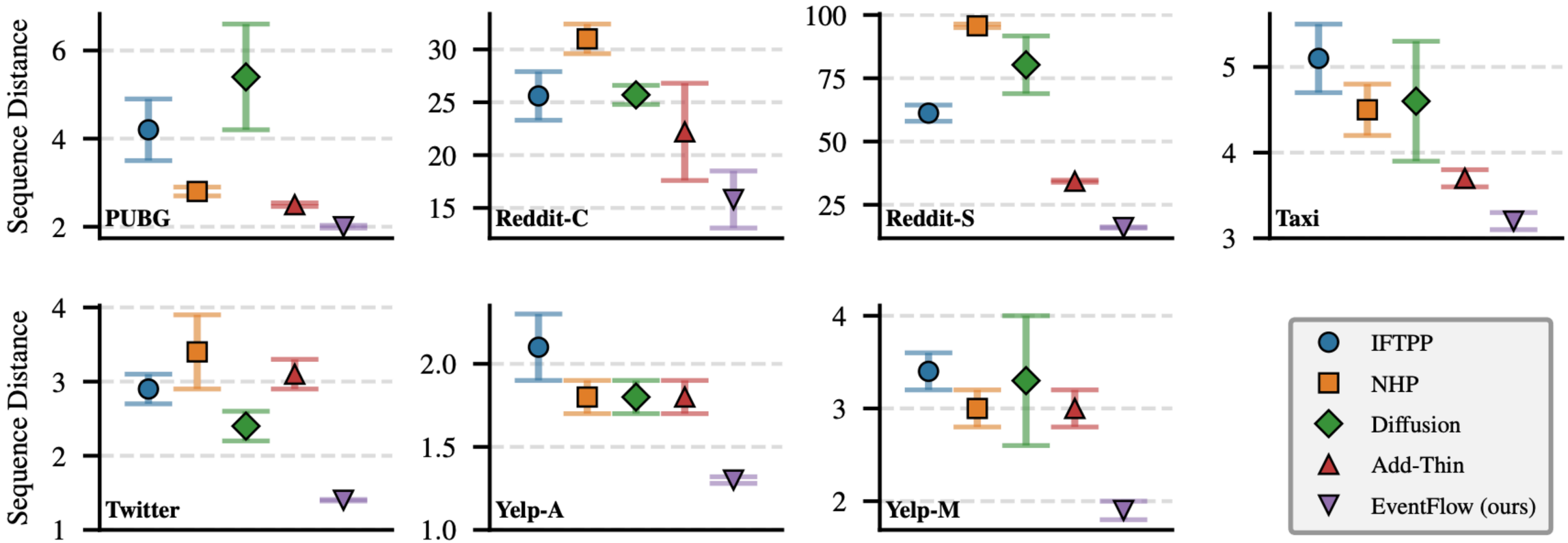
$$d(\mathbf{t}, \hat{\mathbf{t}}) = \sum_{k=1}^{n} \left|t^k - \hat{t}^k\right| + \sum_{k=n+1}^{m} \left(T - \hat{t}^k\right)$$







Experiments: Forecasting Real-World Event Sequences



Isn't this expensive?
Sampling requires solving a neural ODE

For n = 0, 1, … N:

$$t_{s_{n+1}} = v_\theta(s_n, \mathbf{t}_{s_n})\Delta s + t_{s_n}$$

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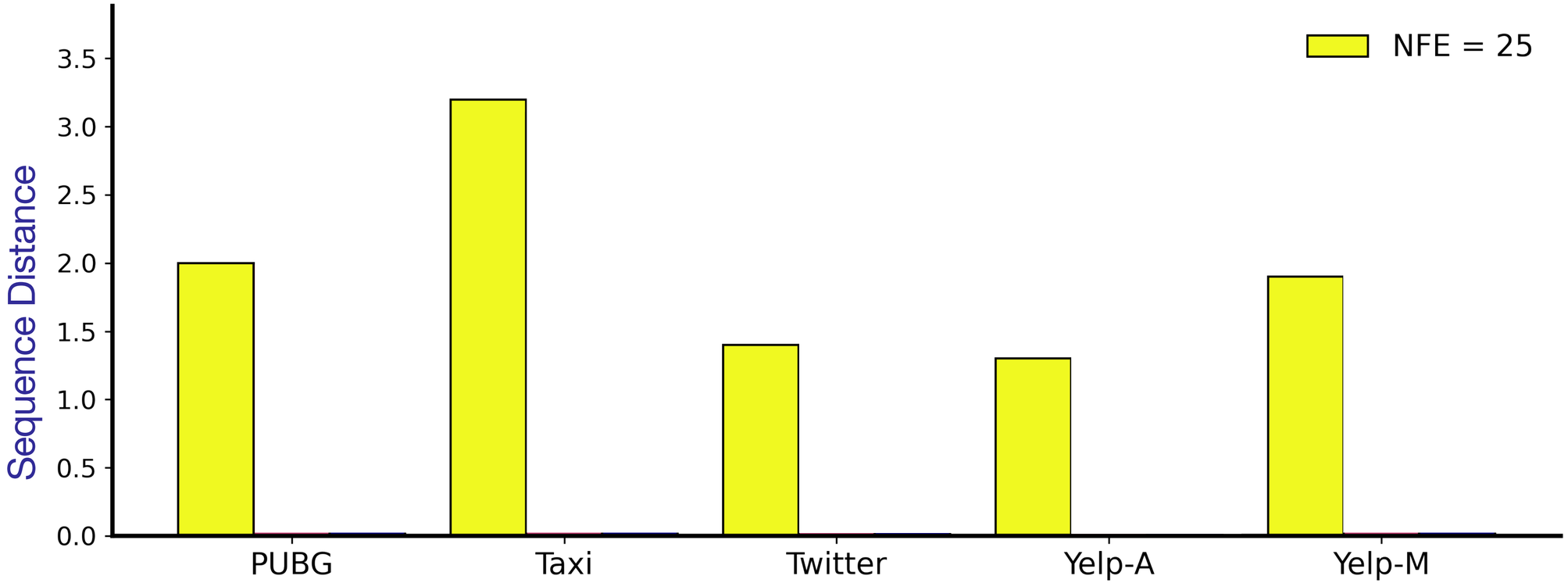

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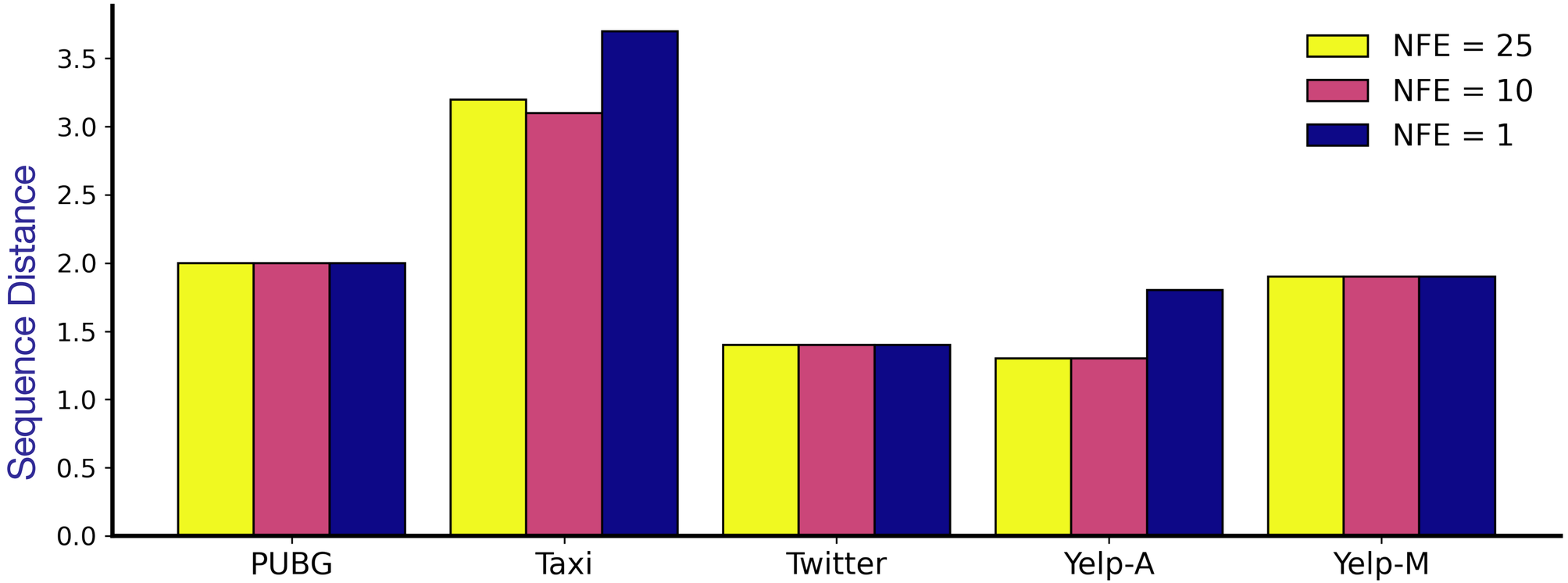

EventFlow is a simple and highly effective method for predicting irregular event sequences

Thank you!

EventFlow: Forecasting Temporal Point Processes with Flow Matching

Gavin Kerrigan[13], Kai Nelson[23], Padhraic Smyth[3]

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