Composable Coresets for Constrained Determinant Maximization and Beyond

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Goal

Design **Composable Coresets**

for **Determinant Maximization**

under **Partition Constraint**

Coresets [Agarwal, Har-Peled, Varadarajan'05]

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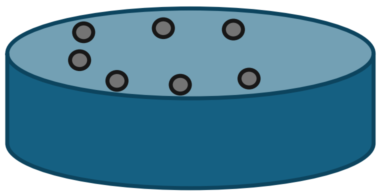

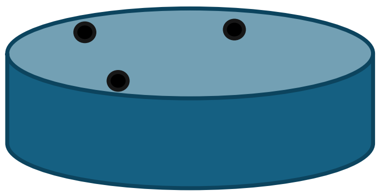

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- Task specific

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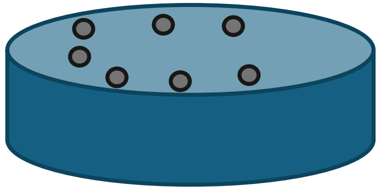
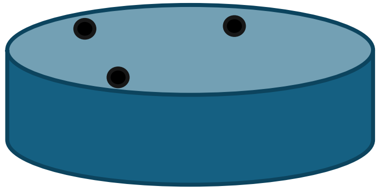
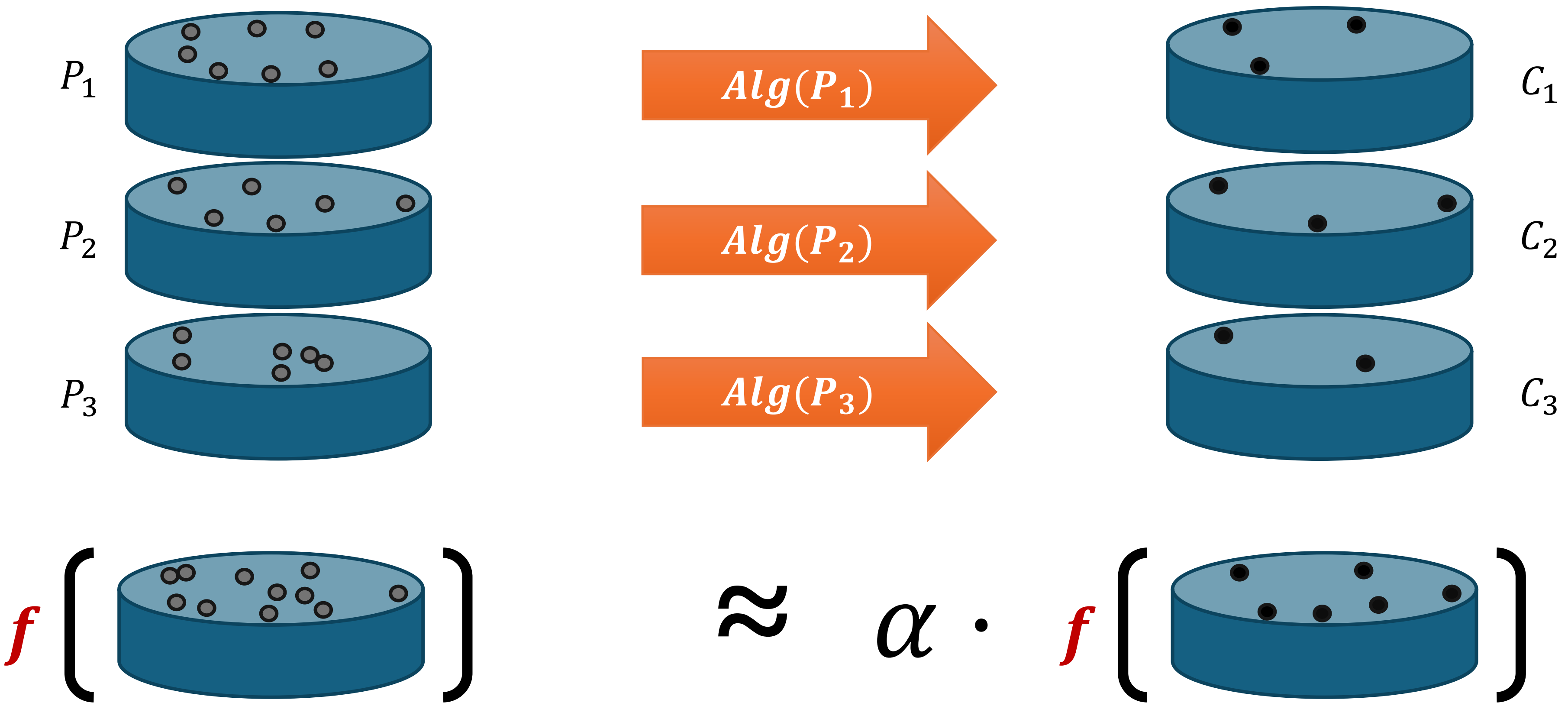
$f($  $)$ $\approx$ $\alpha \cdot$ $f($  $)$

Composable Coresets [Indyk, M., Mahdian, Mirrokni'14]

- **Coresets with composability property:** "The union of coresets is a coreset for the union"
- Summarization Algorithm



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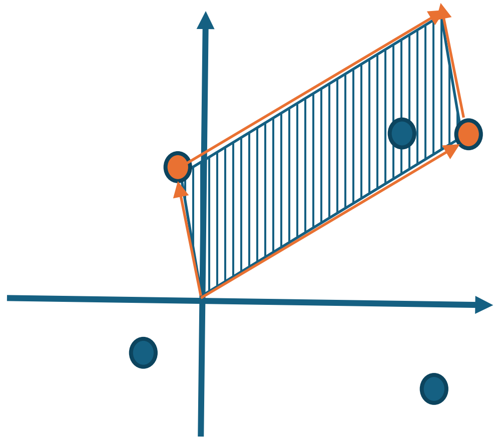

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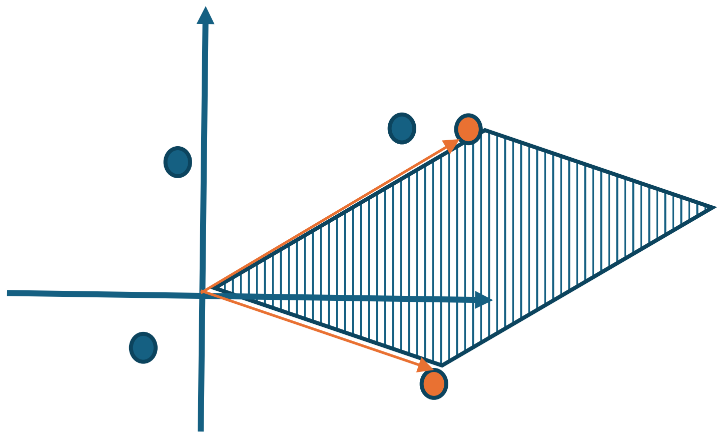

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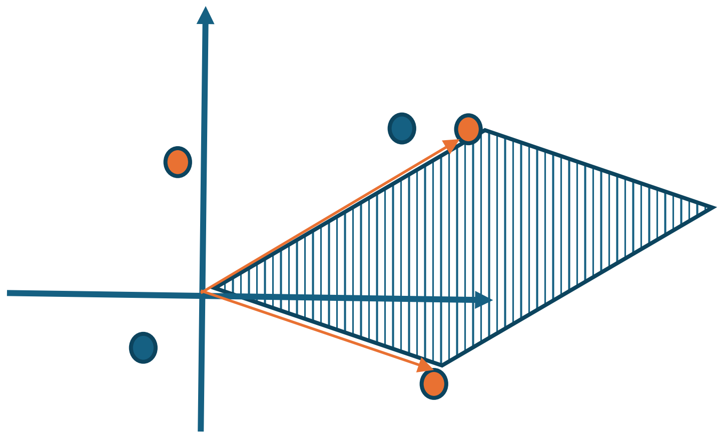

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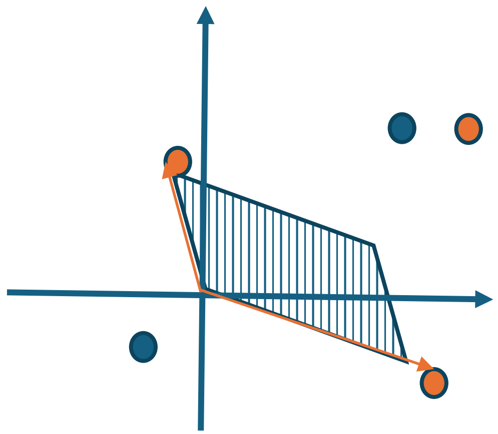

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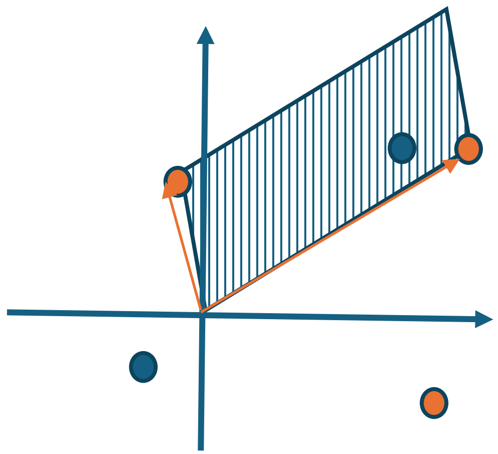

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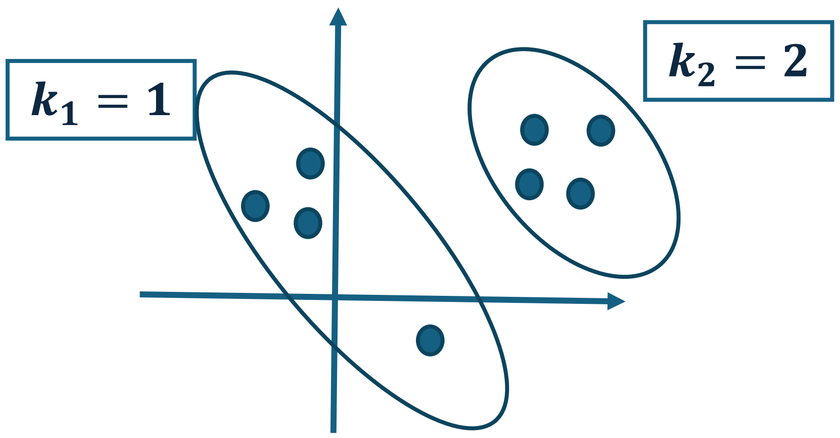

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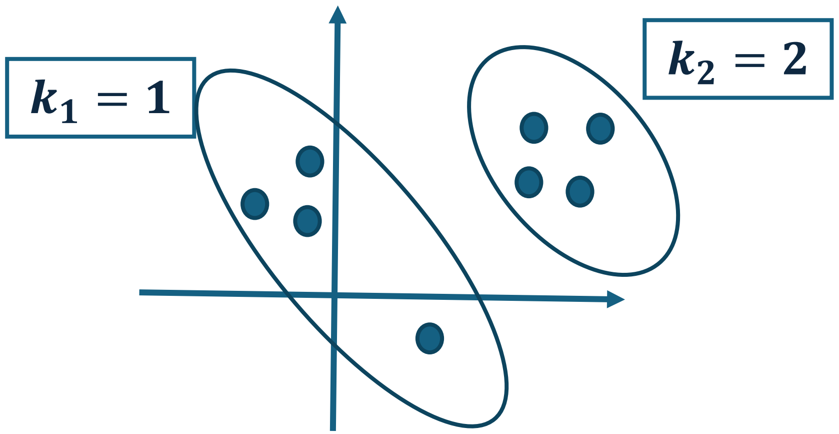

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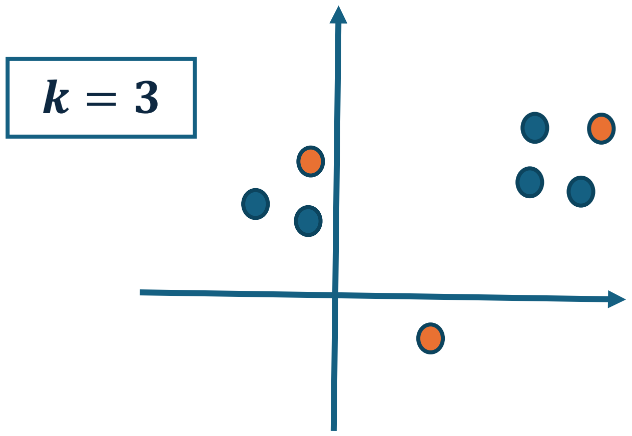

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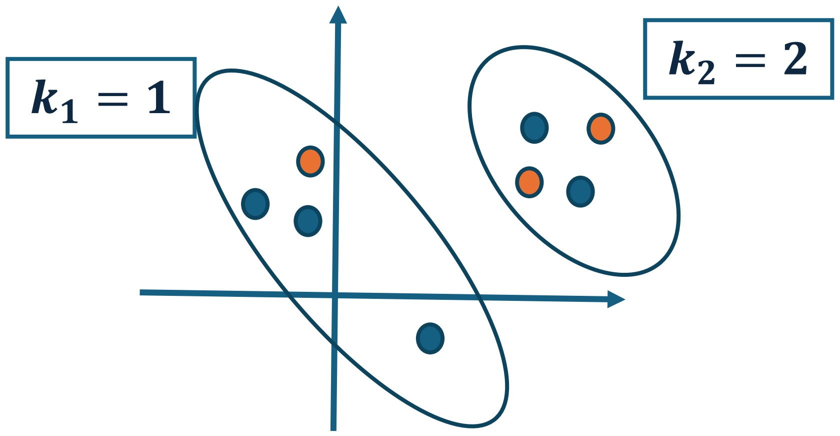

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    1. Different genres of movies in a movie recommendation system, e.g. comedy, action, drama.
    2. Modeling Recency in feed summarization [M., Trajanovski'23]:
        - Each message has a posted time, and the goal is to show more recent messages and less old ones. We still need diversity.
        - Divide the messages in a month into four groups based on the week they have been posted
        - Set k_i to be higher for more recent weeks
    - ...

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- Get new results for the **unconstrained version** of the problem.
- Generalize to other **experimental design** functions
- Generalize to get new results for **Laminar Matroids**
- Generalize to all **Strongly Rayleigh distributions**, obtained via an improved exchange inequality.

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❑ Use to improve the runtime of local-search based algorithm of [Anari-Vuong–COLT'22] for determinant maximization under partition constraint from $O(n^{2^m} k^{2^m})$ to $O(nk^{2^m})$

Thank you!

	Size	Approx.	Constraint
$k \leq d$	k	$O(k)^{2k}$	Cardinality [Indyk et al. 2020, Mahabadi et al. 2019]
$k \geq d$	$\tilde{O}(d)$	$\tilde{O}(d)^{2d}$	Cardinality (with repetition) [Indyk et al. 2020]
$k \geq d$	kd	d^{2d}	Cardinality (without repetition) **This work**
$k \leq d$	mk	k^{2k}	Partition, **This work**
$k \geq d$	kd	d^{2d}	Partition, **This work**
$k \leq d$	k^{2k}	k^{2k}	Laminar, **This work**
$k \geq d$	$(kd)^k$	d^{2d}	Laminar, **This work**