LITE : Efficiently Estimating Gaussian Probability of Maximality

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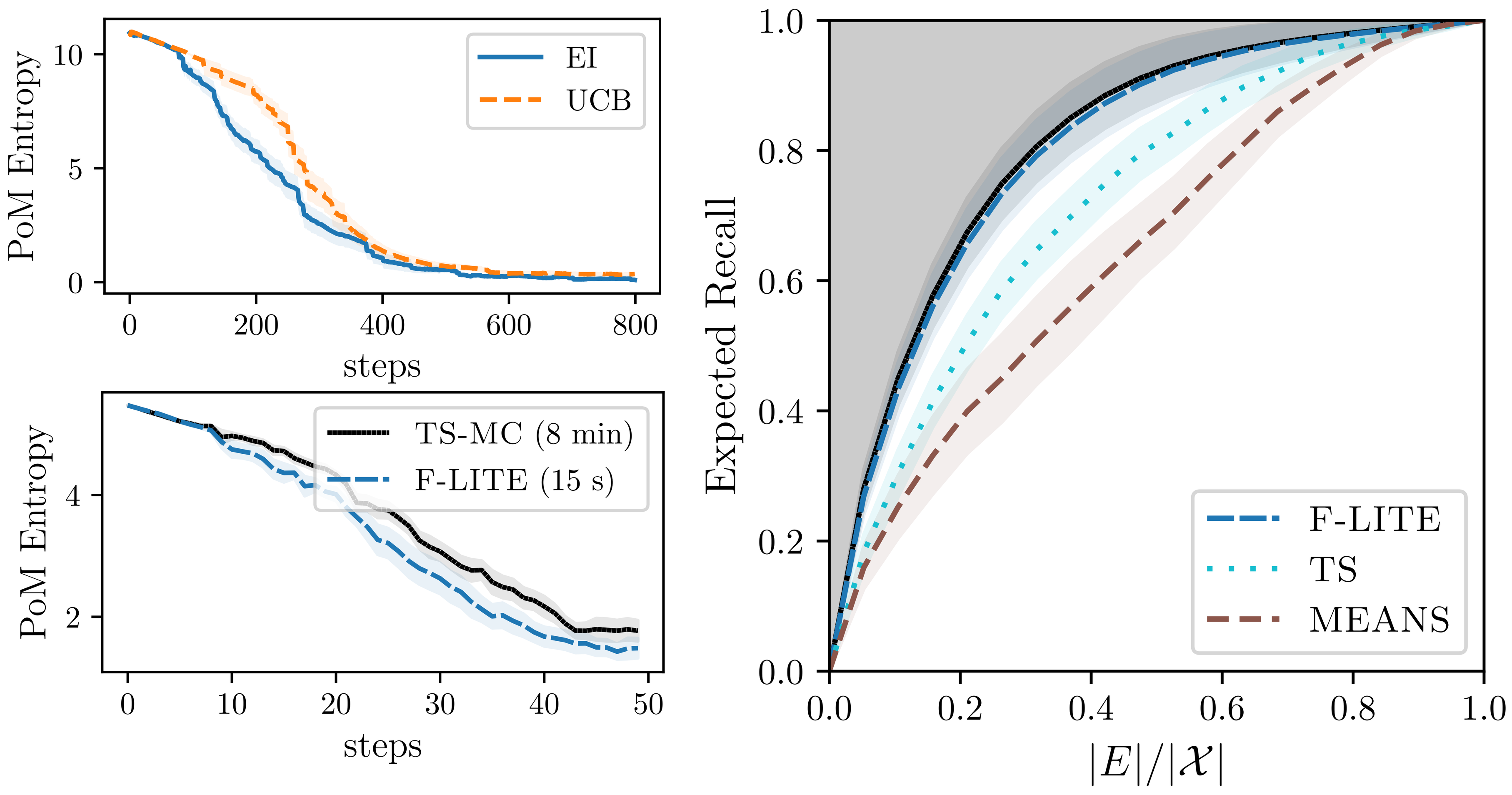
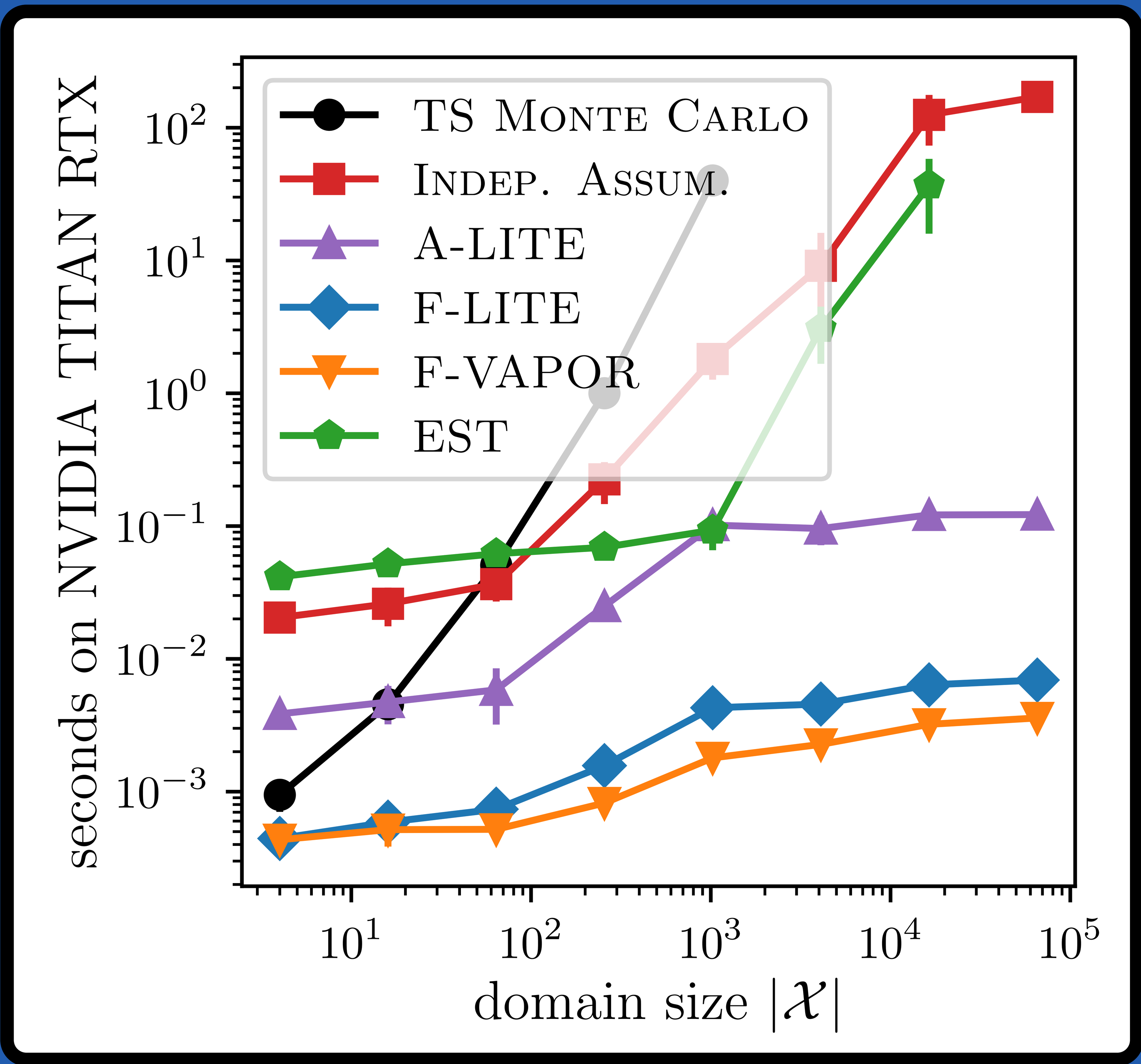
Motivation

The probability of maximality of Gaussian vectors $\mathbb{P}\left[F_x = \max_{z\in\mathcal{X}} F_z\right]$ occurs in Thompson sampling, Entropy Search, entropy estimation, and inverse reinforcement learning, but was poorly understood and very expensive to compute, scaling in $\theta(|\mathcal{X}|^4)$.



An almost-linear time estimator of Gaussian probability of maximality that outperforms prior work in accuracy and runtime.



Method

1. By adopting an *independence assumption* on the Gaussian entries, we simplify from an $|\mathcal{X}|$-dimensional to a one-dimensional integral.

2. To avoid costly numerical integration, we approximate the integrand, which is a CDF, with the CDF of a standard normal and fit m_x and s_x.

$$\tilde{p}_x = \mathbb{P}[\tilde{F}_z \le \tilde{F}_x \ \forall z \neq x] = \mathbb{E}\prod_{z\neq x}\mathbb{P}[\tilde{F}_z \le \tilde{F}_x \mid \tilde{F}_x] \quad (1)$$

$$\approx \mathbb{E}\Phi\left(\frac{\tilde{F}_x - m_x}{s_x}\right) = \Phi\left(\frac{\mu_{F_x} - m_x}{\sqrt{\sigma_{F_x}^2 + s_x^2}}\right) \quad (2)$$

- A-LITE uses quartile matching to fit the free parameters m_x and s_x.
- F-LITE sets $s_x = 0$ (extreme-value theorem) and uses the normalization condition to find $m_x = \kappa^*$.

Theoretical Insights

Proposition 4. *Define the variational objective*

$$\mathcal{W}(p) := \sum_{x\in\mathcal{X}} p_x \cdot \left(\mu_{F_x} + \underbrace{\sqrt{2\tilde{I}(p_x)}\cdot\sigma_{F_x}}_{\text{exploration bonus}}\right), \quad (5)$$

with the quasi-surprisal $\tilde{I}(u) := (\phi(\Phi^{-1}(u))/u)^2/2$. *Then the maximizer of* $\mathcal{W}$ *among elements of the probability simplex is given by* F-LITE, *i.e., by* q *with*

$$q_x := \Phi\left(\frac{\mu_{F_x} - \kappa^*}{\sigma_{F_x}}\right) \quad \text{with } \kappa^* \text{ s.t. } \sum_x q_x = 1.$$

TV-Distance	Synthetic Distributions	1-dim GP	2-dim GP (E.2)	DropWave (E.3)	Quadcopter
EST	11.54 ± 0.20	45.6 ± 2.7	15.1 ± 1.2	5.17 ± 0.64	14.3 ± 2.0
VAPOR	9.89 ± 0.11	37.0 ± 2.0	15.7 ± 1.0	5.70 ± 0.72	17.2 ± 2.5
F-LITE (ours)	4.65 ± 0.08	$\mathbf{13.7 \pm 1.0}$	10.9 ± 0.7	$\mathbf{4.87 \pm 0.60}$	11.1 ± 1.4
A-LITE (ours)	$\mathbf{3.76 \pm 0.06}$	$\mathbf{14.1 \pm 1.0}$	$\mathbf{7.5 \pm 0.5}$	$\mathbf{4.32 \pm 0.53}$	$\mathbf{8.7 \pm 0.9}$
Indep. Assum.	0.00 ± 0.00	6.7 ± 0.4	6.6 ± 0.2	3.85 ± 0.54	9.0 ± 1.0