Robust Training in High Dimensions via Block Coordinate Geometric Median Descent

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Robust DNN Training

- Training DNN involves optimizing over highly over-parameterized , non-convex loss landscape.

- (Gross Corruption) Adversary can replace $0 \leq \psi \leq 1/2$ fraction of them with ***arbitrary*** points. If G and B are sets of good and bad points $\alpha = \frac{|B|}{|G|} = \frac{\psi}{\psi - 1} \leq 1$

- *smooth non-convex* problems with finite sum structure, under gross corruption, *without any prior knowledge about the malicious samples*.

$$\min_{\mathbf{x} \in \mathbb{R}^d} \left[f(\mathbf{x}) := \frac{1}{|\mathbb{G}|} \sum_{i \in \mathbb{G}} f_i(\mathbf{x}) \right]$$

SGD under gross corruption

- SGD proceeds as follows : $\mathbf{x}_{t+1} := \mathbf{x}_t - \gamma \tilde{\mathbf{g}}^{(t)}, \quad \tilde{\mathbf{g}}^{(t)} = \frac{1}{|\mathcal{D}_t|} \sum_{i \in \mathcal{D}_t} \nabla f_i(\mathbf{x}_t).$

- Even a single corrupt sample can lead SGD to an *arbitrarily poor solution*.
  - This can be attributed to the *linear gradient aggregation* step.

- Breakdown Point: smallest fraction of contamination that must be introduced to cause an estimator to produce arbitrarily wrong estimates.

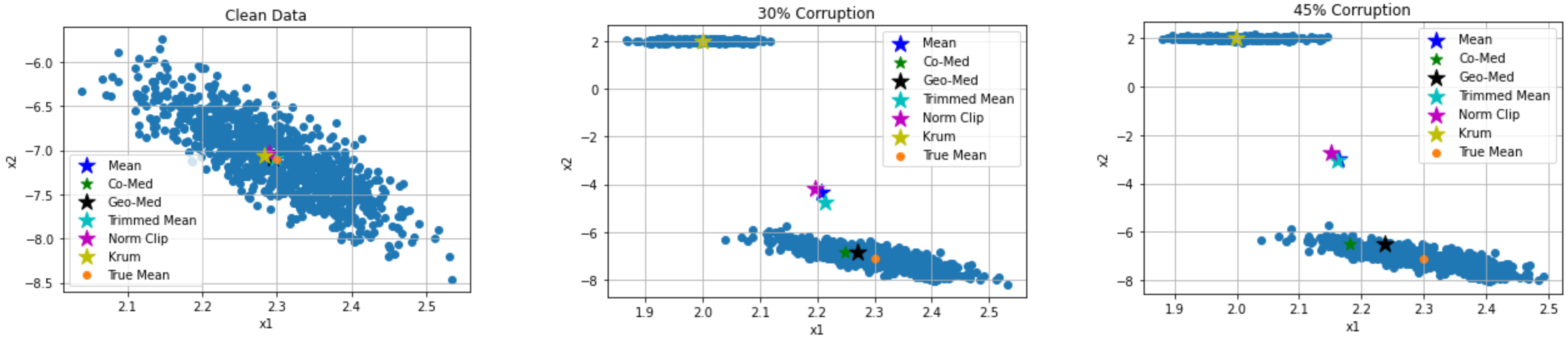
- SGD has lowest possible *asymptotic breakdown of 0* under gross corruption. Consider a single malicious gradient: $\mathbf{g}_j^{(t)} = -\sum_{i \in \mathcal{D}_t \setminus j} \mathbf{g}_i^{(t)}$

Robust Gradient Aggregation

- Make SGD Robust Again: Replace Mean with *Robust Mean Estimator*

- Geometric Median: $$\mathbf{x}_* = \textsc{Gm}(\{\mathbf{x}_i\}) = \arg\min_{\mathbf{y}\in\mathbb{X}} \left[g(\mathbf{x}) := \sum_{i=1}^{n} \|\mathbf{y} - \mathbf{x}_i\| \right]$$

- *Achieves Optimal Breakdown point of ½ .*



This Toy example in 2 dimensions demonstrates the superior robustness properties of GM for estimating the aggregated gradient even in presence of heavy corruption.

Geometric Median Descent

- GM-SGD : $x_{t+1} = \mathrm{x_t} - \eta\hat{g}_t$, $\hat{g}_t = \mathrm{GM}(\{\mathrm{g_i}\})$

- Unfortunately, finding GM is computationally hard.

- Best known algorithm to find $\epsilon -$ approximate GM i.e. $g(\mathbf{x}) \leq (1+\epsilon)g(\mathbf{x}_*)$ of n points in R^d requires $O(d/\epsilon^2)$.

- GM-SGD is computationally intractable for optimization in high dimensions arising from DNN e.g. $d \approx$ 60M Alexnet, $d \approx$ 175B GPT3

Block Coordinate GM Descent

- DNNs are over-parameterized
  - Performing *gradient aggregation in low dimensional subspace* should have *little impact* in the downstream optimization task.

- Judiciously *subset a block of k dimensions* (k << d) and perform GM in R^k
  - Ideally, select k dimensions resulting in largest decrease loss - NP Hard ☹
  - Select k columns with largest total norm from

- k << d can imply *large information loss* resulting in *slower convergence*.
  - Keep track of Residual and add back to gradient estimate.
  - Fixes sampling bias and retains convergence.

Theoretical Guarantee

- Non-convex and Smooth : Suppose f_i corresponding to non-corrupt samples i.e. $i \in G$ are L smooth and non-convex. Run BGMD with ϵ approximate GM oracle and $\gamma = \frac{1}{2L}$ in presence of α corruption for T iterations. Sample any iteration τ uniformly at random then:

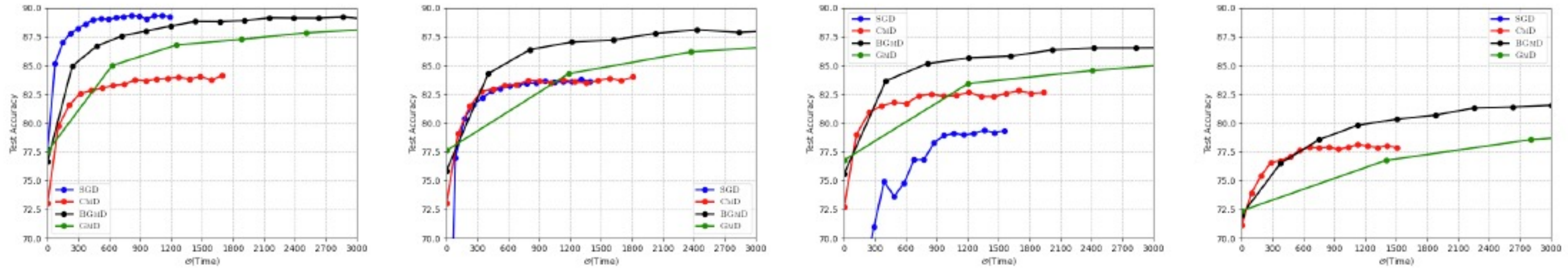
$$\mathbb{E}\|\nabla f(\mathbf{x}_\tau)\|^2 = \mathcal{O}\left(\frac{LR_0}{T} + \frac{\sigma^2\xi^{-2}}{(1-\alpha)^2} + \frac{L^2\epsilon^2}{|\mathbb{G}|^2(1-\alpha)^2}\right)$$

Algorithm	Aggregation Operator*	Iteration Complexity†	Breakdown Point†‡
SGD	MEAN(·)	$\boldsymbol{\mathcal{O}(bd)}$	0
(Yang et al., 2019; Yin et al., 2018)	CM(·)	$\mathcal{O}(bd\log b)$	**1/2**
(Wu et al., 2020)	GM(·)	$\mathcal{O}(d\epsilon^{-2}+bd)$	**1/2**
BGmD (This work)	BGM(·)	$\mathcal{O}(k\epsilon^{-2}+bd)$	**1/2**
(Data and Diggavi, 2020)	(Steinhardt et al., 2017)	$\mathcal{O}(db^2\min(d,b)+bd)$	1/4
(Blanchard et al., 2017)	KRUM(·)	$\mathcal{O}(b^2d)$	$\lfloor\beta\rfloor$
(Yin et al., 2018)	CM$_\beta$(·)	$\mathcal{O}(bd(1-2\beta)+bd\log b)$	$\lfloor\beta\rfloor$
(Ghosh et al., 2019; Gupta et al., 2020)	NC$_\beta$(·)	$\mathcal{O}(bd(2-\beta)+b\log b)$	$\lfloor\beta\rfloor$

Empirical Evidence : Feature Corruption

- Feature Corruption Simulation
  - *Huber's Contamination*: $z_t \sim \mathcal{N}(100, 1)$ directly added to the images.
  - *Impulse Corruption:* Salt and Pepper noise added by setting 90% of pixels to 0 or 1.
  - *Gaussian Blur:* Kernel size (5,5) and $\sigma = 100$.

Top (L: Clean, R: Huber's Contamination). Bottom(L: Impulse, R: Gaussian Blur).



(a) No corruption (b) 10% Corruption (c) 20% Corruption (d) 40% Corruption

Test accuracy as a ***function of wall clock time*** for training Fashion-MNIST using LeNet (1.16 M params) in presence of impulse noise.