
# Are All Linear Regions Created Equal?

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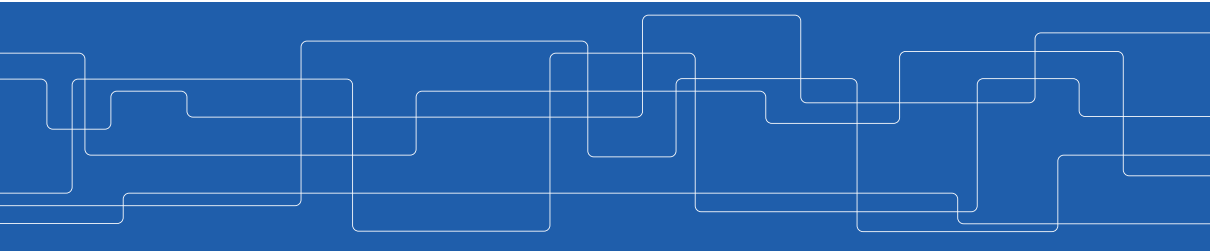



# Research question

- ReLU networks express continuous piece-wise affine functions of their input variable $\mathbf{x}$.
- To capture expressivity, interpreted as nonlinearity, estimating the number of linear "pieces" – i.e. the *density* of linear regions – has been proposed.
- However, merely *counting* linear regions heavily assumes that they all meaningfully contribute nonlinearity.
- Is this the case for standard overparameterized models?

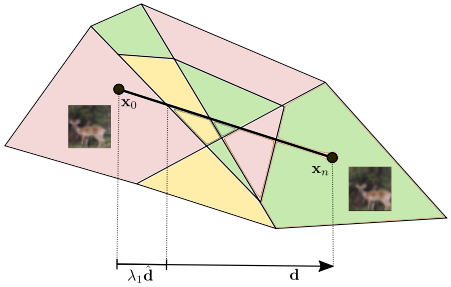

# Contributions

So far, studies of linear regions were limited to small networks and MLPs.

1. We propose a fast linear region discovery algorithm, that scales to realistic networks.
2. We introduce a principled measure on nonlinearity for ReLU networks.
3. We study ConvNets and ResNets trained in practice on the CIFAR datasets.
4. We show that density is an unreliable predictor in practice.
5. Our measure of nonlinearity follows the test error in a model-wise deep double descent regime.

# Linear Region Discovery

- Iterative algorithm, given endpoints $\mathbf{x}_0, \mathbf{x}_n$ and a direction $\mathbf{d} = \mathbf{x}_n - \mathbf{x}_0$
- Find $\lambda$ to cross the closest linear region boundary along $\mathbf{d}$.
- We use the algorithm to study nonlinearity in proximity of the "data manifold".



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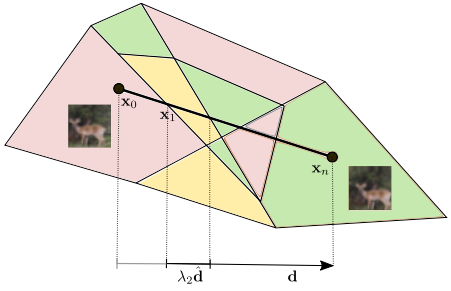
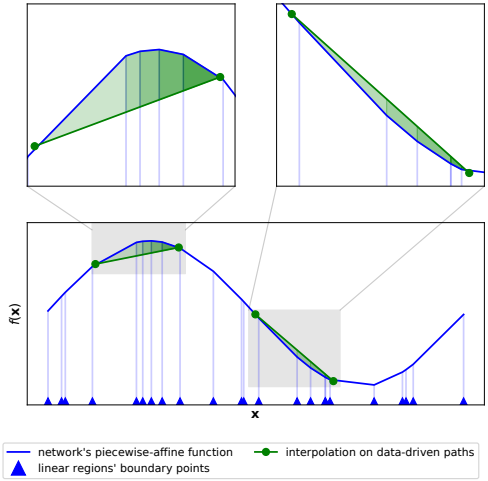


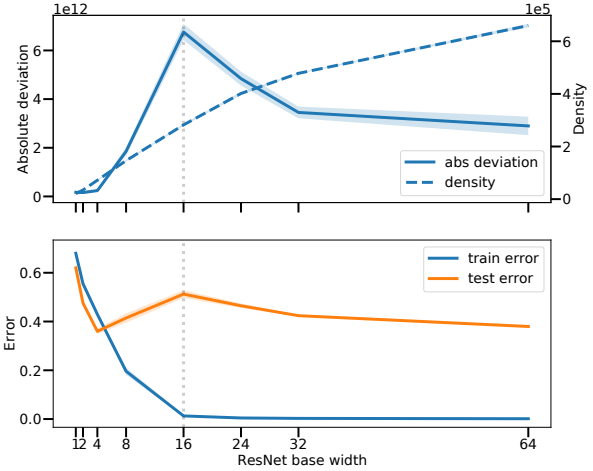
# Measuring Expressivity

- Study variation of the function expressed by the network.
- Measure of how far the network deviates from a simple affine function $\mathbf{a}(\mathbf{x})$.
- $\mathbf{a}(\mathbf{x})$ interpolates the network's function at different training points / augmentations.



# Reduced Nonlinearity

- Density increases with model size.
- Absolute deviation follows model-wise deep double descent.
- Past the noise interpolation threshold, local redundancy of linear regions increases.
- Nonlinearity is reduced, with improved test error!



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## Code

https://bit.ly/linear-regions