

Fixed Support Tree-Sliced Wasserstein Barycenter

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Background

Wasserstein Barycenter

Wasserstein Distance

- Time complexity: $O(M^2)$

$$W_d(\mathbf{a}, \mathbf{b}) = \min_{T \in \Pi(\mathbf{a}, \mathbf{b})} \langle \mathbf{T}, \mathbf{C} \rangle$$

Wasserstein Barycenter

- Generative models [Simon+ 2020]
- Topic model [Xu+ 2018]
- **Time complexity:** $O(NM^2)$

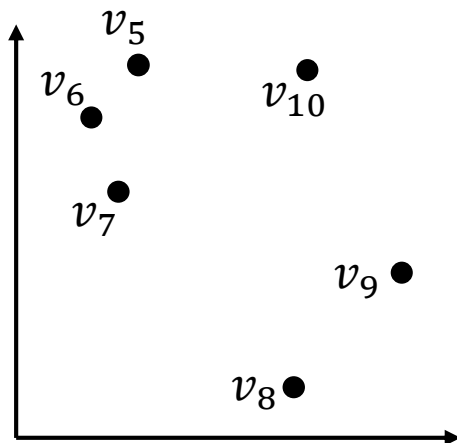
N : #samples, M : #supports

Background

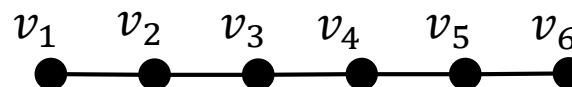
(Tree-)Sliced Wasserstein Distance

Wasserstein Distance

- $O(M^2)$

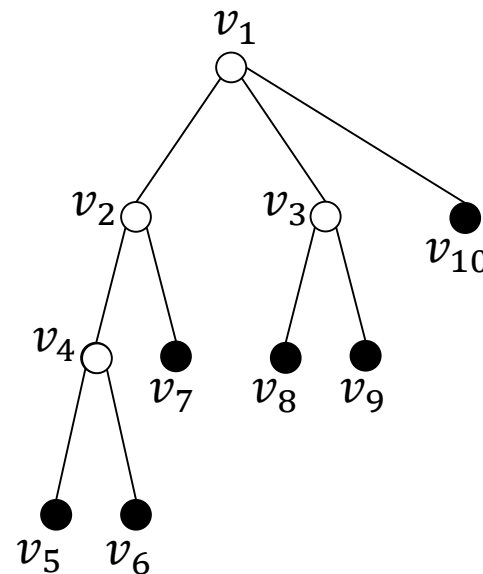


Sliced-Wasserstein Distance



Tree-Sliced Wasserstein Distance

- Closed-form solution, $O(M)$



M : #supports

Contribution

Fixed Support Tree-Sliced Wasserstein Barycenter (FS-TSWB)

We introduce the **Fixed Support Tree-Sliced Wasserstein Barycenter**.

- Wasserstein barycenter on a tree metric.

Proposed Algorithms:

	Time Complexity
FS-WB (IBP [Benamou+15])	$O(M^2N)$
FS-TSWB (PSD)	$O(T(\log M + N + D)M)$
FS-TSWB (FastPSD)	$O(T(\log M + \log N + D)M)$

Experimentally:

- The FS-TSWB can be solved 2-order magnitude faster than the FS-WB.

T : #slices, N : #samples, M : #supports, $D(\ll M, N)$: the depth of a tree

Proposed Method

Fixed Support Tree-Wasserstein Barycenter

- **Tree-Wasserstein Distance** [Takezawa+2021]

$$W_{d_T}(\mathbf{a}_i, \mathbf{a}_j) = \|\mathbf{B}\mathbf{a}_i - \mathbf{B}\mathbf{a}_j\|_1$$

- **Fixed Support Tree-Wasserstein Barycenter (FS-TWB)**

$$\operatorname{argmin}_{\mathbf{a} \in A} \frac{1}{N} \sum_{i=1}^N \|\mathbf{B}\mathbf{a} - \mathbf{B}\mathbf{a}_i\|_1$$

- Convex optimization.
- **Projected Subgradient Descent (PSD)** is applicable.
- We propose more efficient algorithm called **FastPSD**

A : simplex

Experimental Results

Time Consumption

- The FS-TSWB can be computed faster than the FS-WB.

Table 1: Time consumption [seconds].

		MNIST	AMAZON	AGNews
Proposed method	FS-WB	64.4	2129.6	10811.7
	FS-TSWB ($T = 1$)	5.2	62.4	86.1
	FS-TSWB ($T = 5$)	25.1	330.7	449.4
	FS-TSWB ($T = 10$)	51.1	653.9	899.9
	FS-TSWB ($T = 15$)	78.9	969.5	1346.6
	FS-TSWB ($T = 20$)	111.9	1287.3	1788.4
	FS-TSWB ($T = 25$)	142.6	1610.3	2236.4

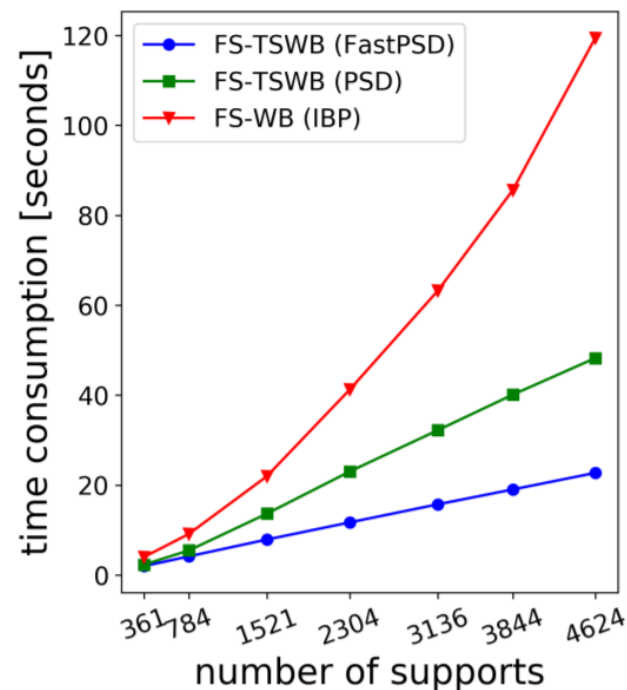
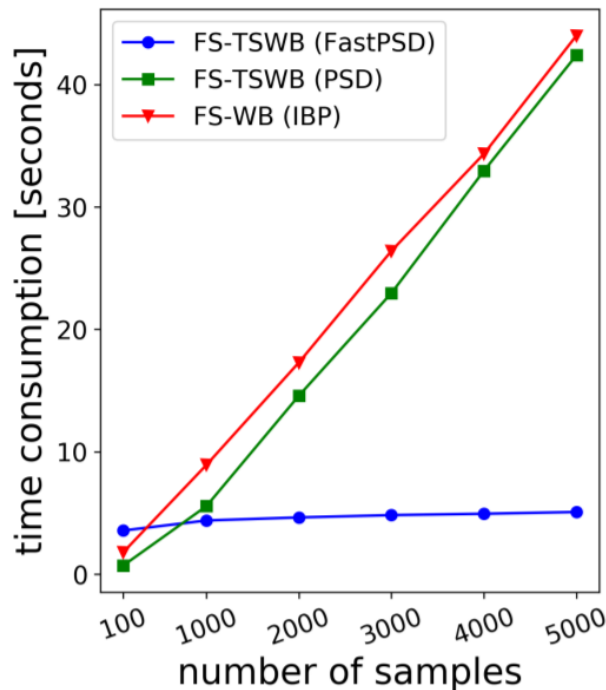


T : #slices

Experimental Results

Time Consumption (MNIST)

- The FS-TSWB can be computed faster than the FS-WB in terms of both the number of samples and the number of supports.



Experimental Results

Visualization

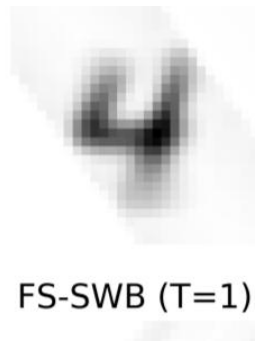
- The FS-TSWB is closer to the FS-WS than the FS-SWB.

Wasserstein Barycenter



FS-WB

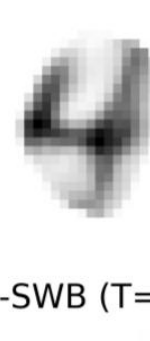
Sliced-Wasserstein Barycenter



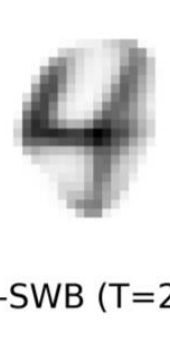
FS-SWB (T=1)



FS-SWB (T=5)



FS-SWB (T=10)



FS-SWB (T=25)

Tree-Sliced Wasserstein Barycenter



FS-TSWB (T=1)



FS-TSWB (T=5)



FS-TSWB (T=10)



FS-TSWB (T=25)