

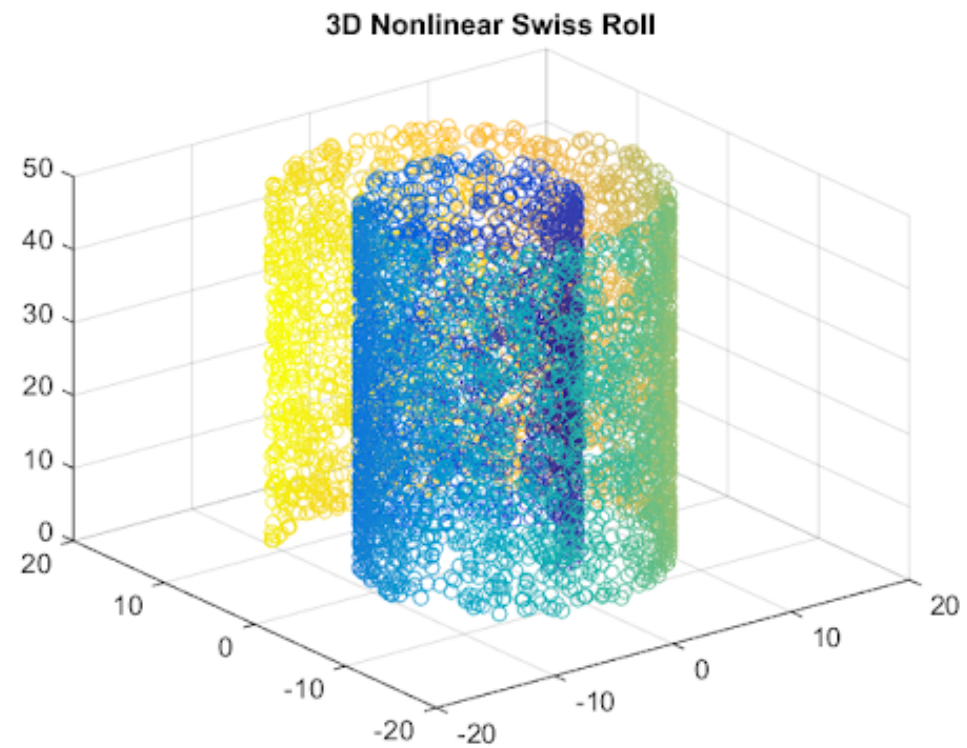
# Product Manifold Learning

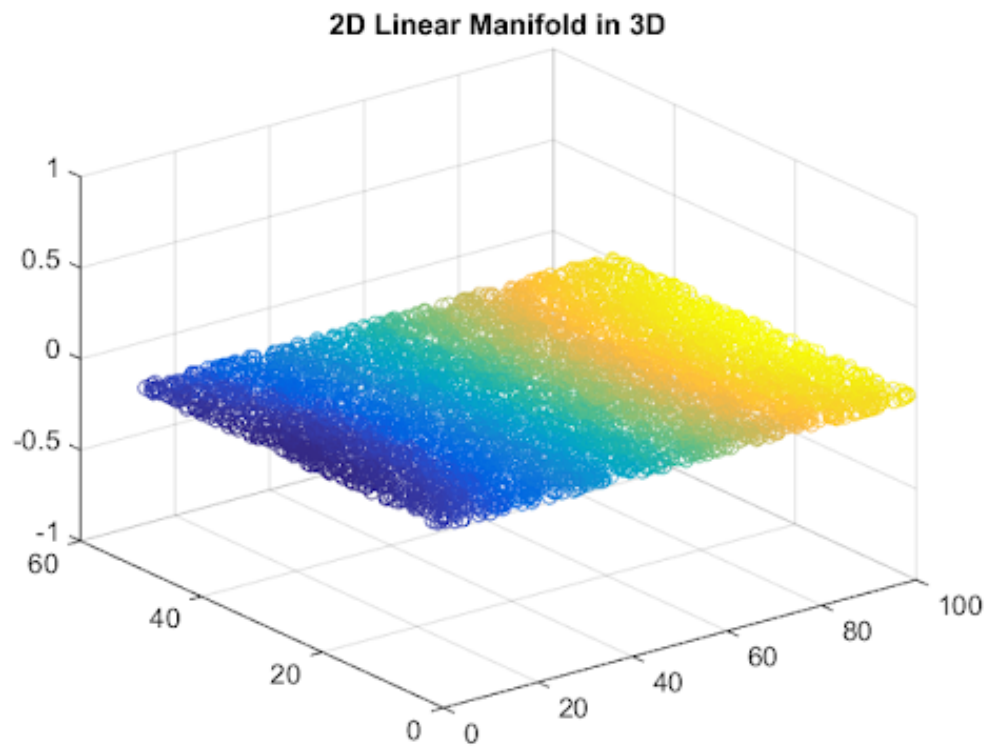
Sharon Zhang, Amit Moscovich, Amit Singer

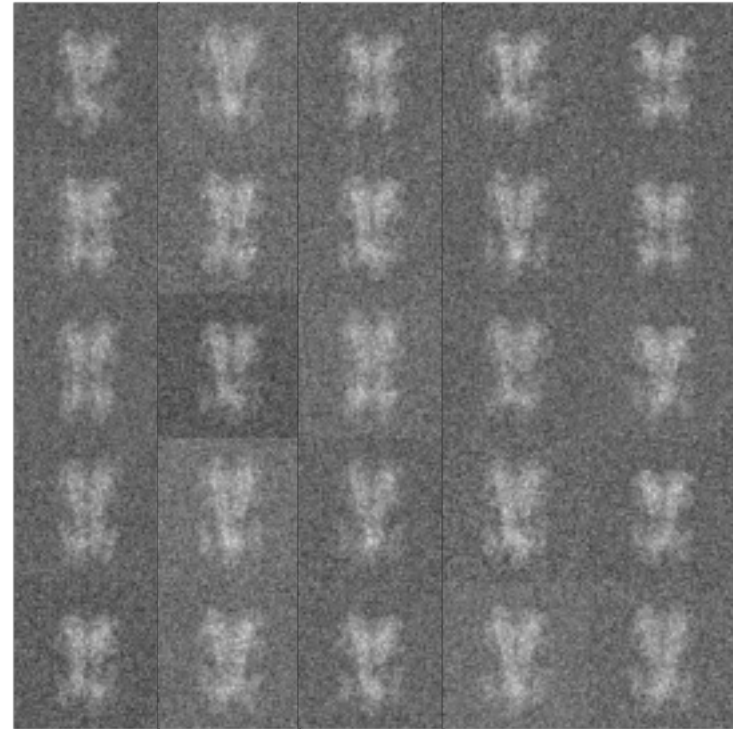
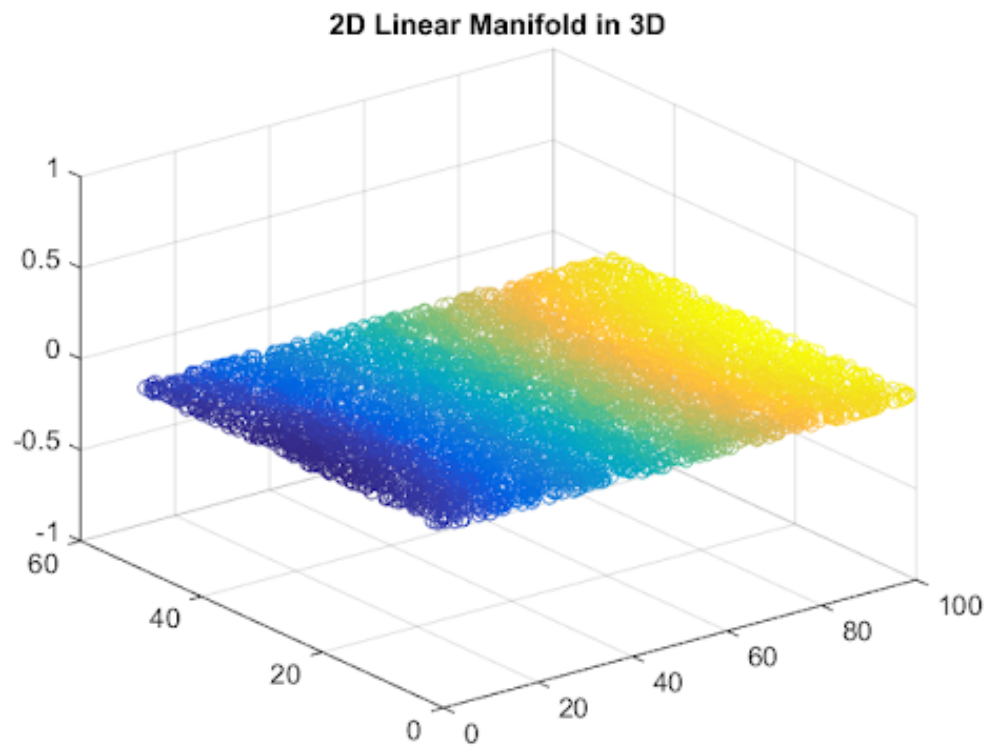
Princeton University

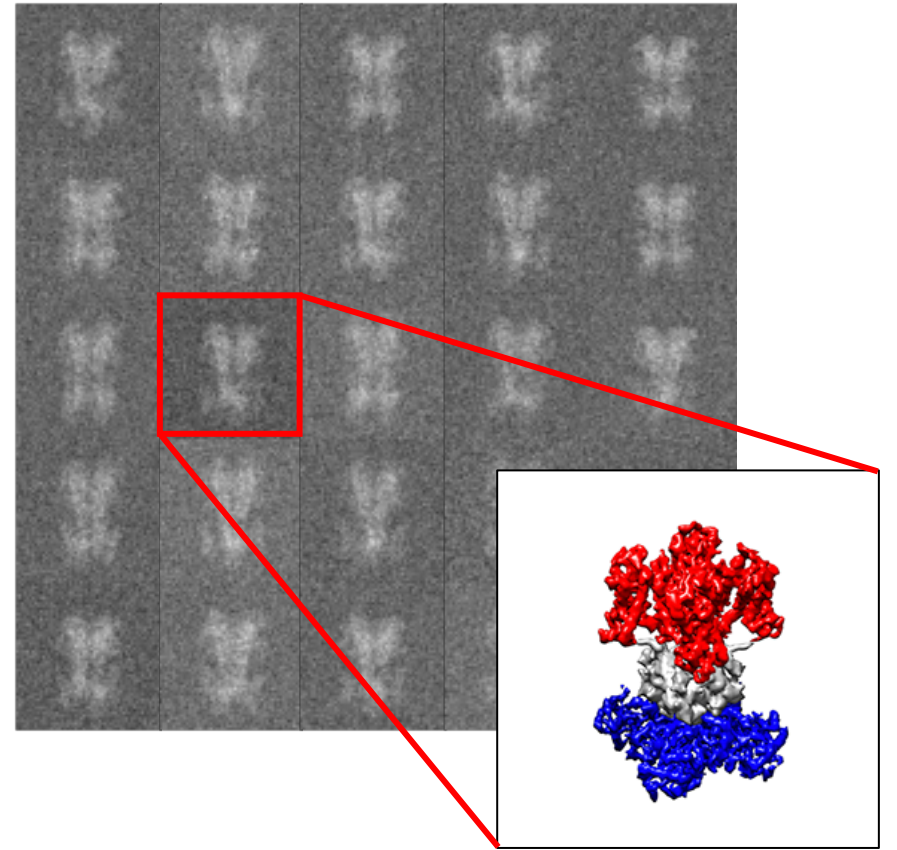
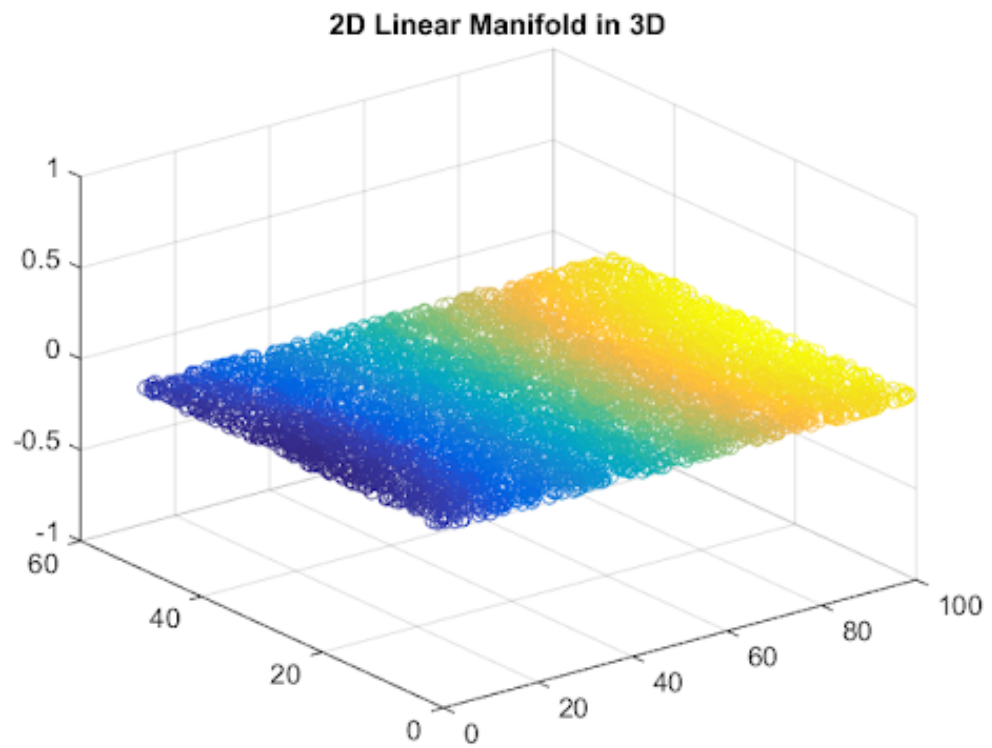
AISTATS 2021











# Product manifold learning

$$F : \mathcal{M} \rightarrow \mathbb{R}^D, \quad (\theta_1, \dots, \theta_m) \mapsto \mathbf{x}$$

$$\mathcal{M} = \mathcal{M}_1 \times \dots \times \mathcal{M}_m.$$

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manifold factorization

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manifold factors

manifold factorization



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manifold factors

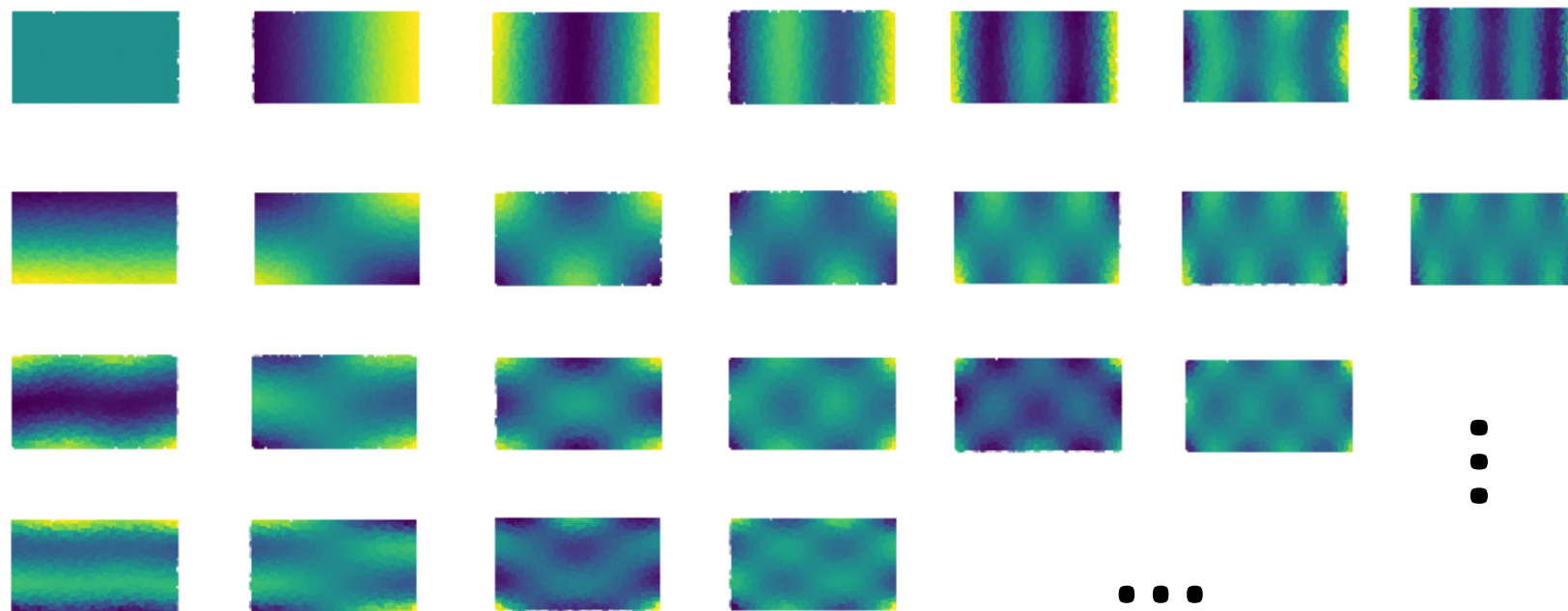
manifold factorization

How can we factorize (or disentangle)  $\theta_1, \dots, \theta_m$ ?

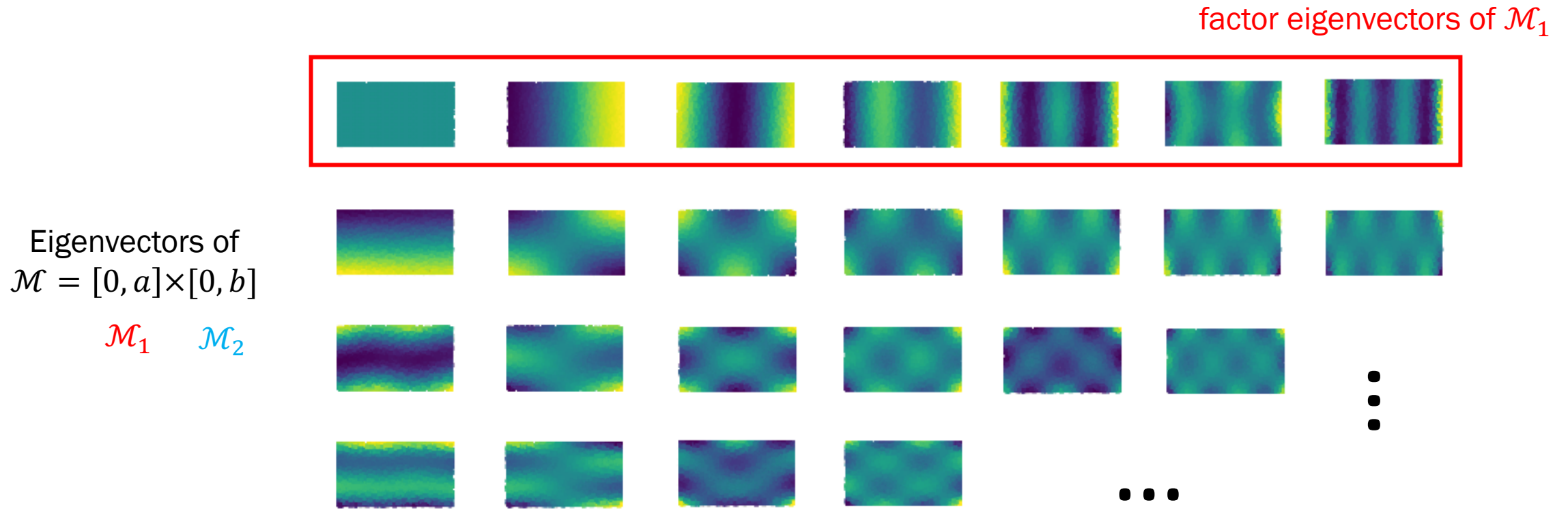
# Separability of Laplacian

Eigenvectors of  
 $\mathcal{M} = [0, a] \times [0, b]$

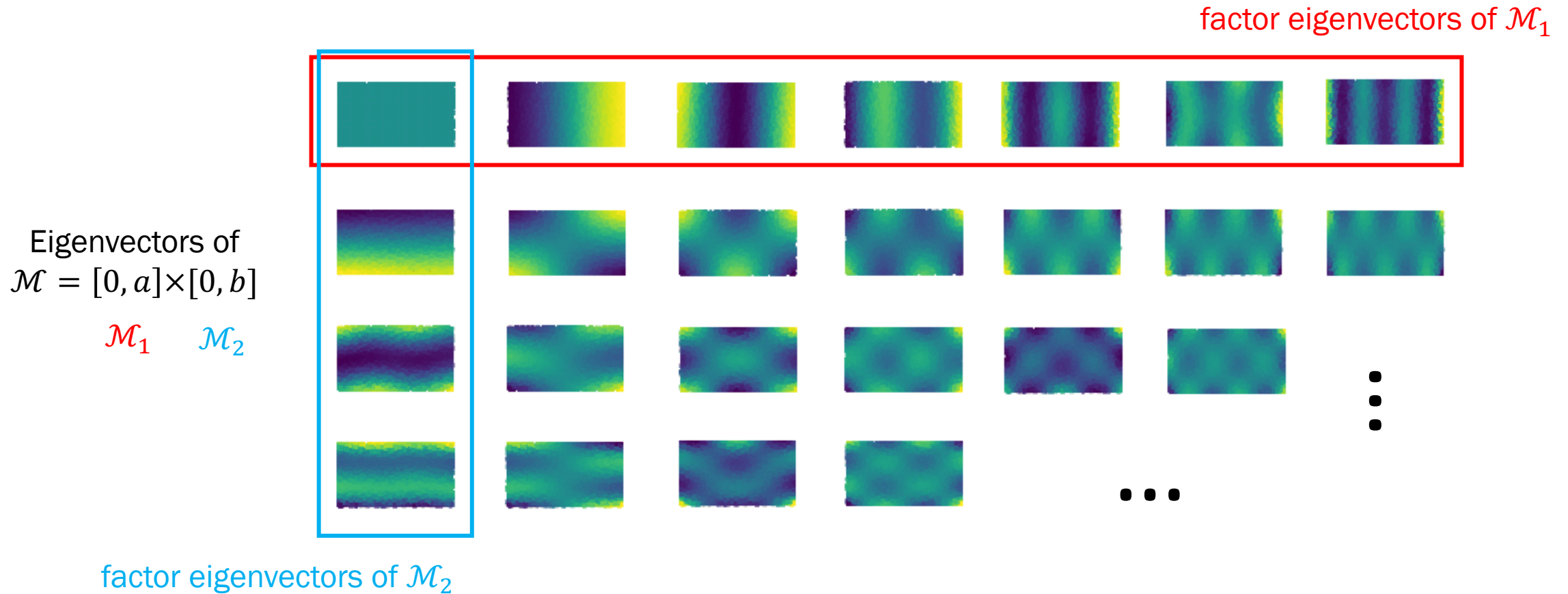
$\mathcal{M}_1$   $\mathcal{M}_2$



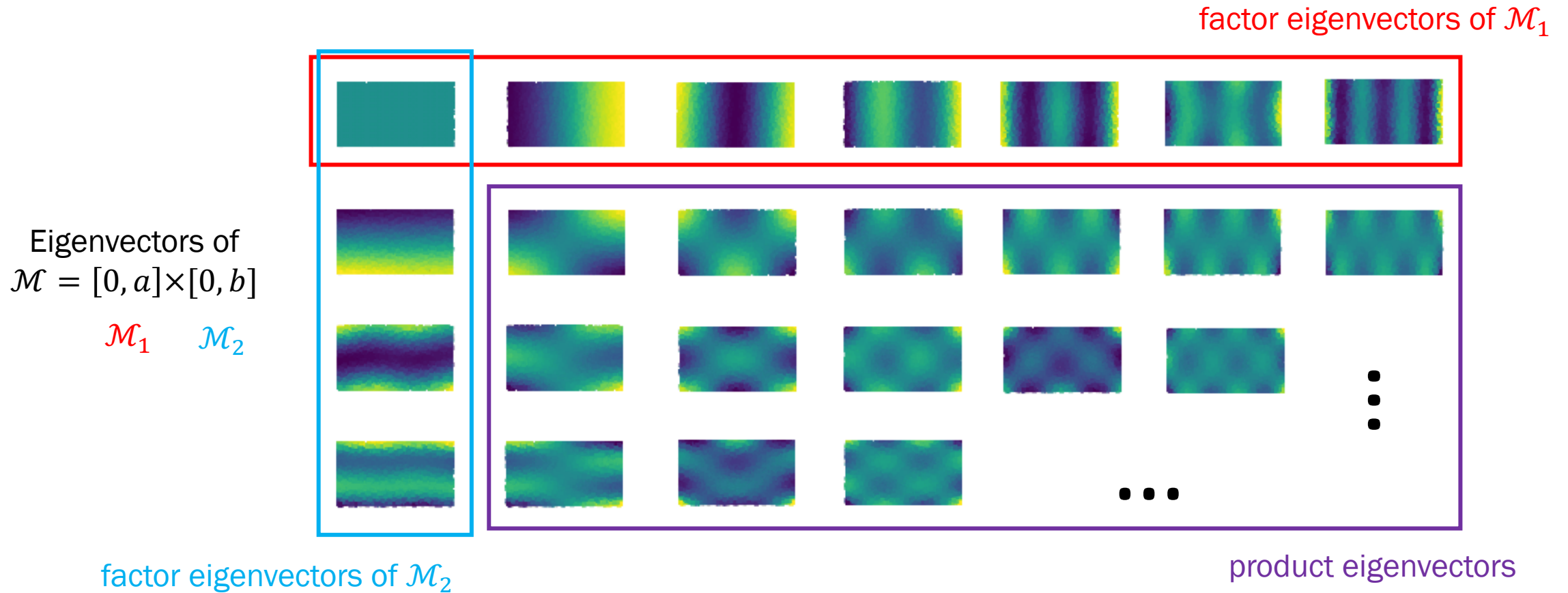
# Separability of Laplacian



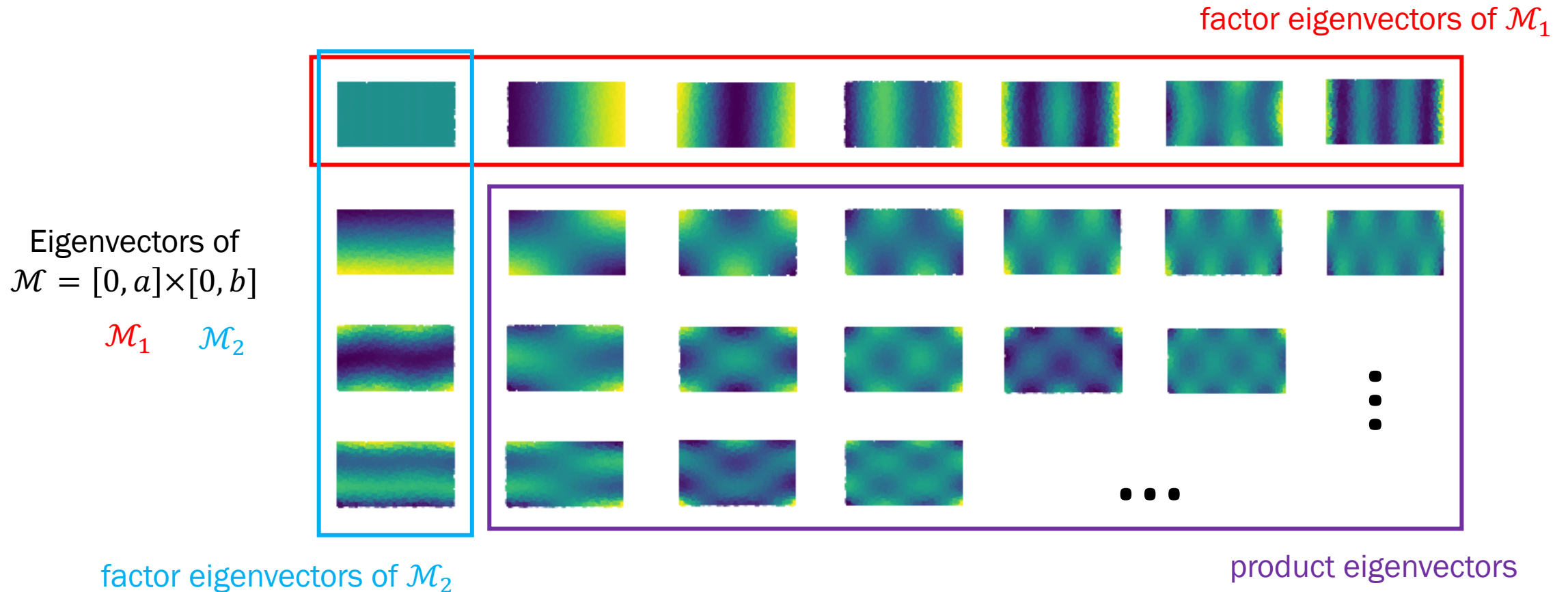
# Separability of Laplacian



# Separability of Laplacian

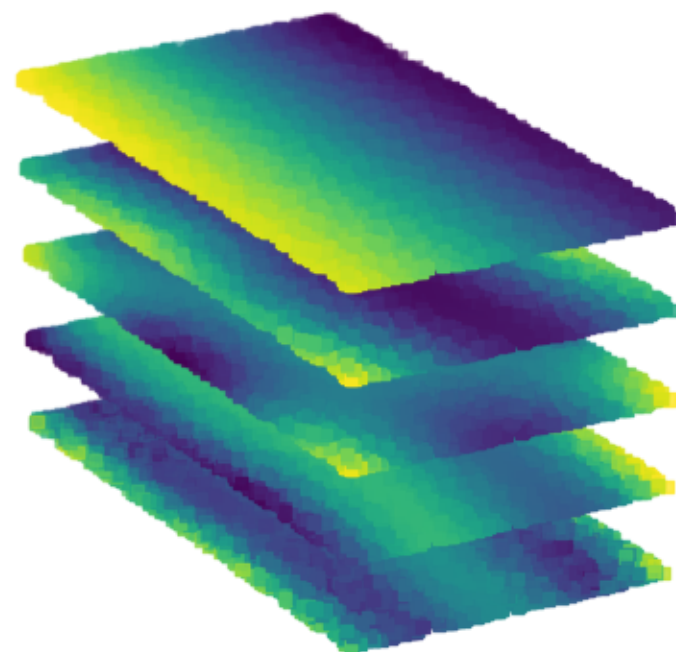
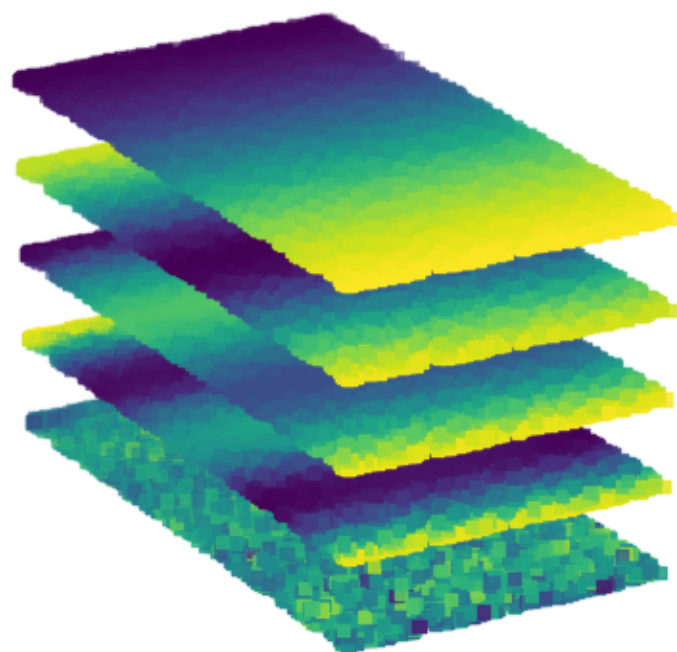
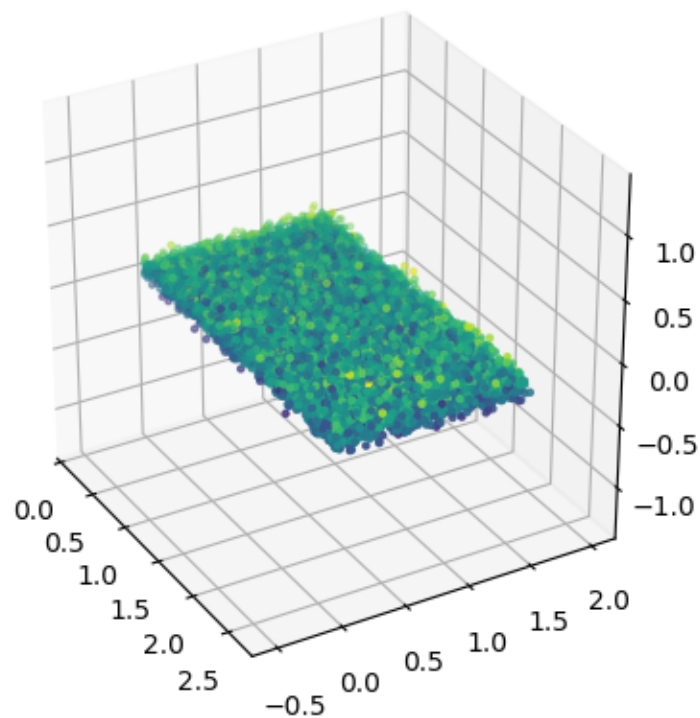


# Separability of Laplacian



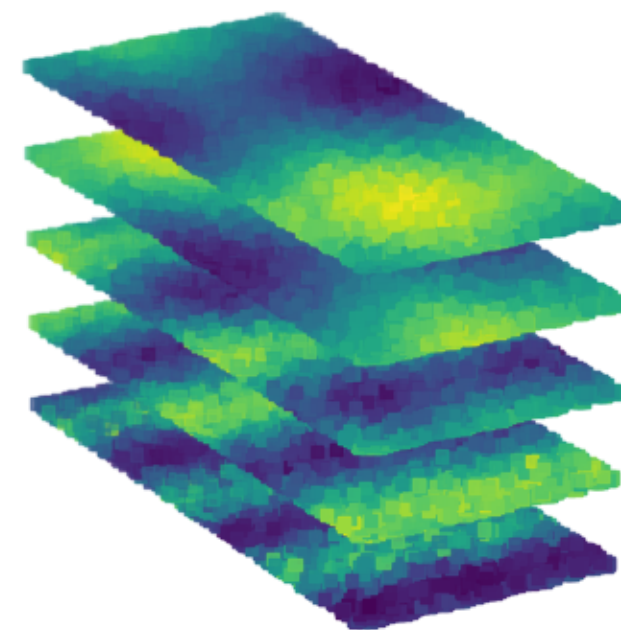
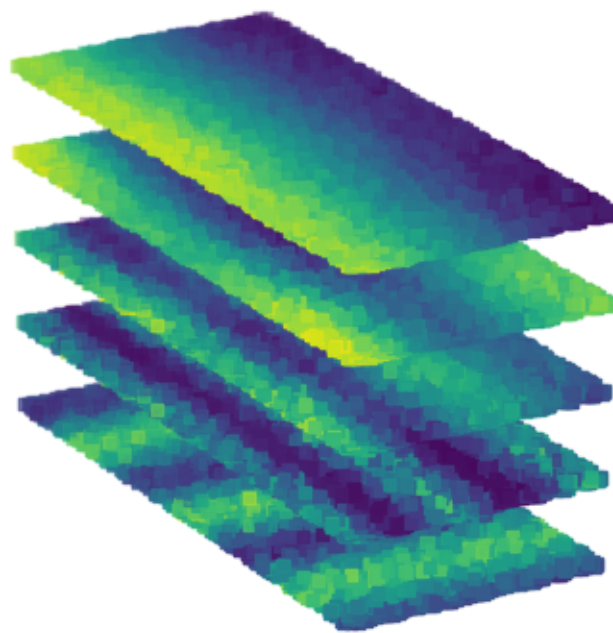
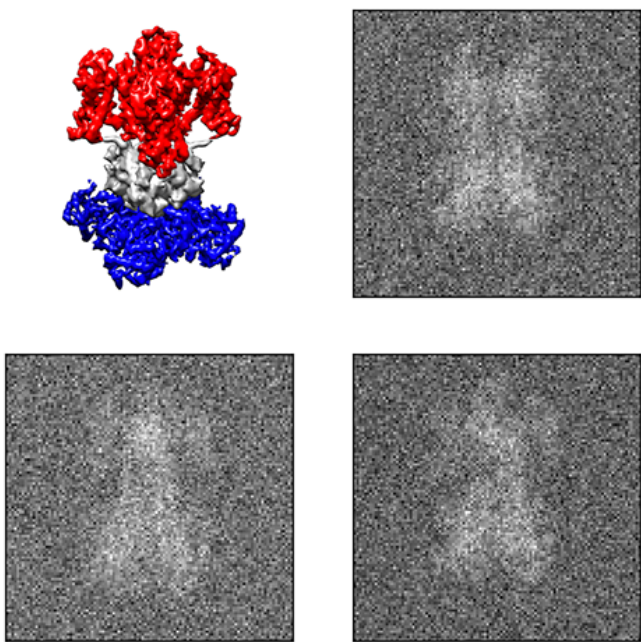
See Yaiza Canzani's 2013 lecture notes, "Analysis on Manifolds via the Laplacian" [4].

# 3D rectangle

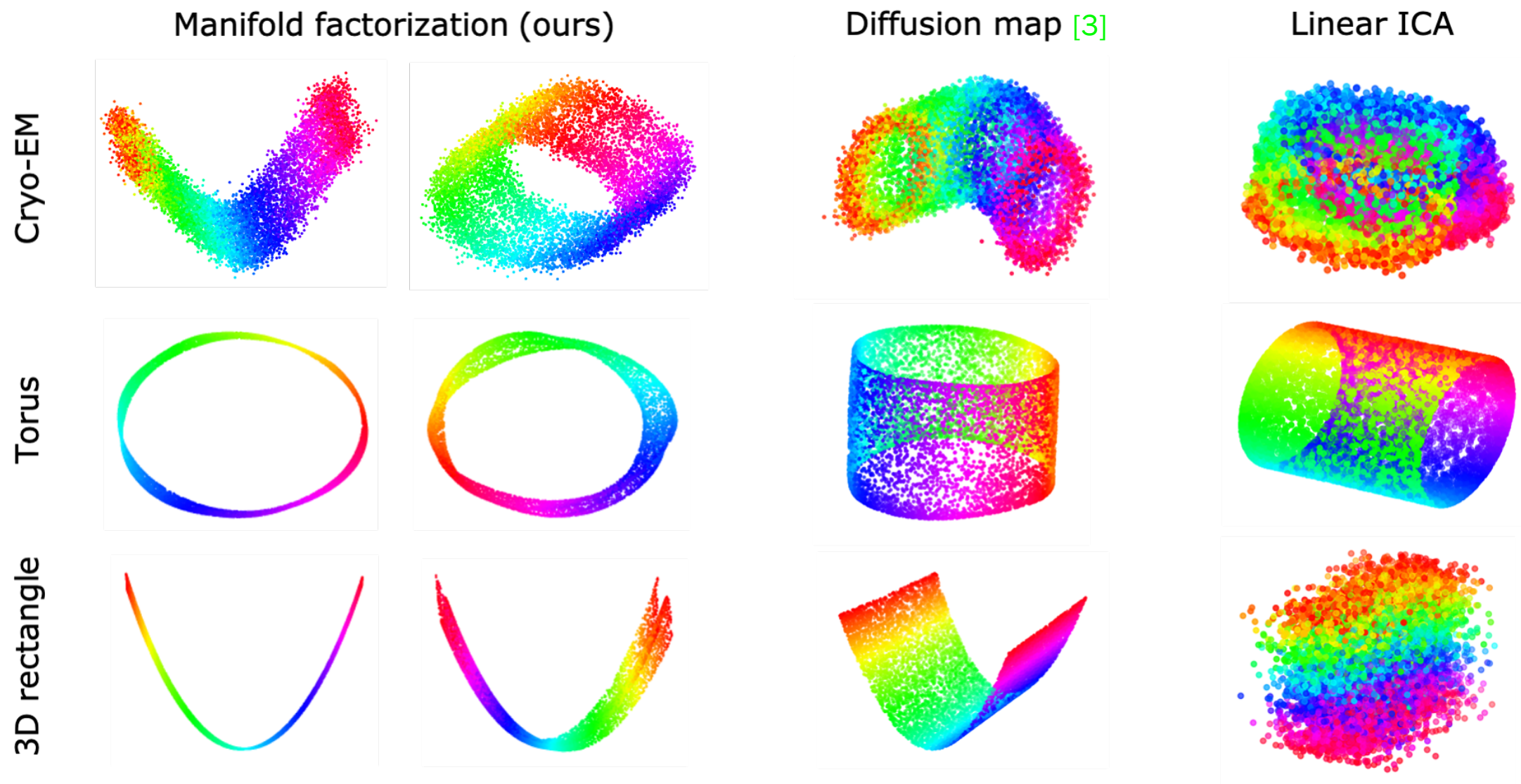


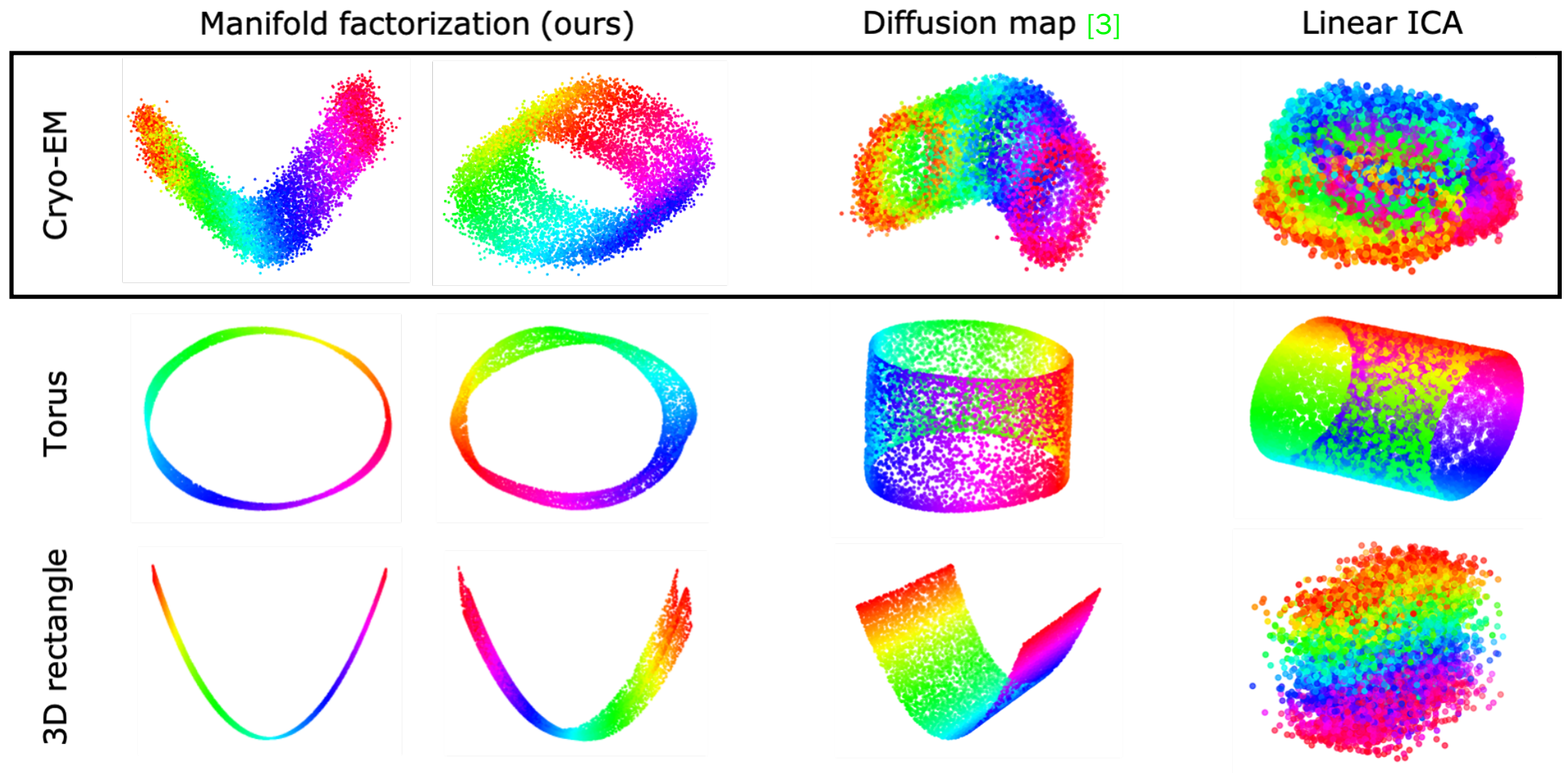


# Cryo-EM data









# Thank you!

## References

- [1] Sharon Zhang, Amit Moscovich, Amit Singer. **Product Manifold Learning**. *AISTATS 2021*.
- [2] Mikhail Belkin and Partha Niyogi. Towards a theoretical foundation for Laplacian-based manifold methods. *Journal of Computer and System Sciences*, 74 (8):1289–1308, 2008.
- [3] Ronald R. Coifman and Stéphane Lafon. Diffusion maps. *Applied and Computational Harmonic Analysis*, 21(1):5–30, 2006.
- [4] Yaiza Canzani. Analysis on Manifolds via the Laplacian (lecture notes), 2013.

## Code

<https://github.com/sxzhang25/product-manifold-learning>