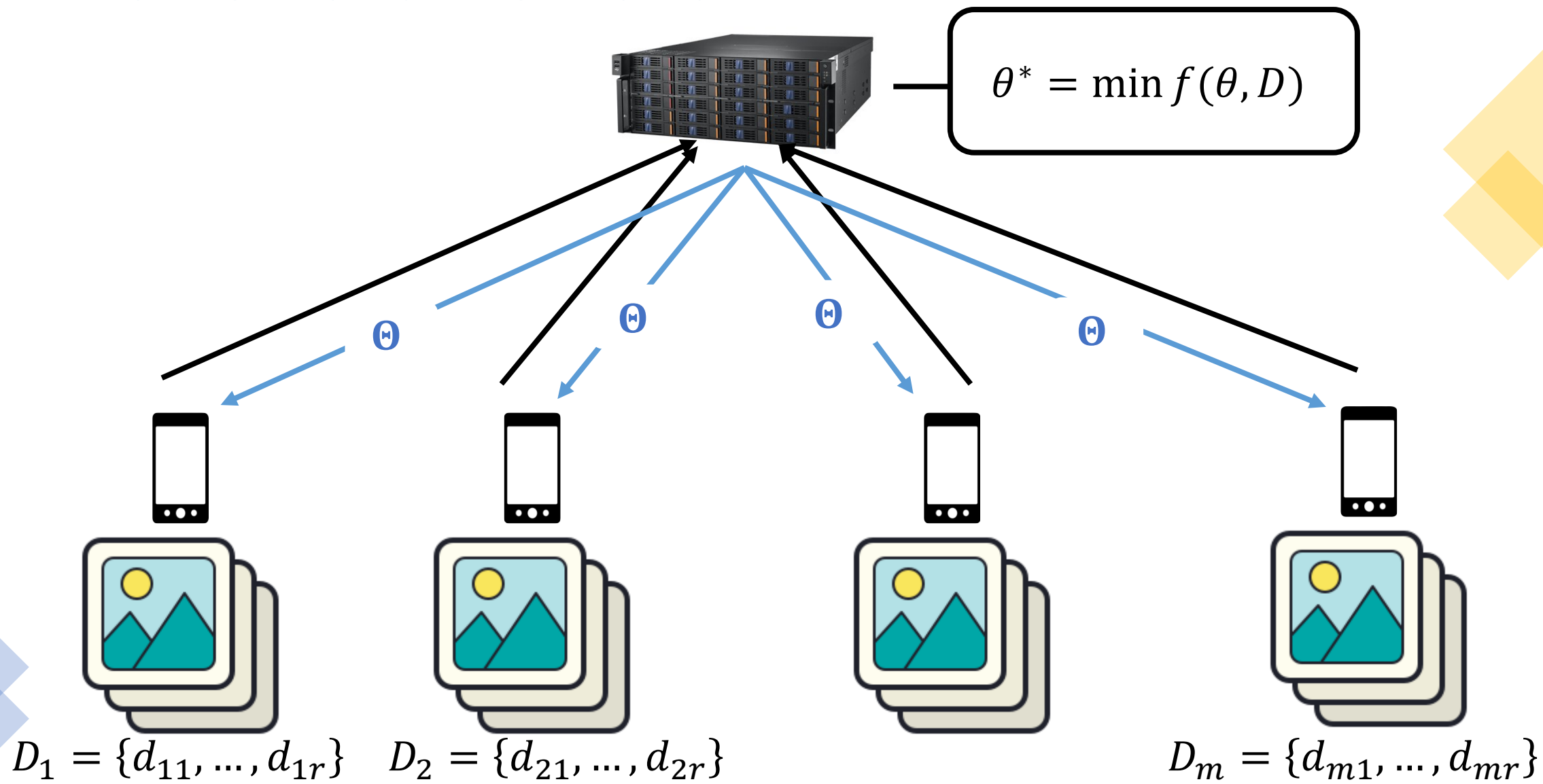




# Shuffled Model of Differential Privacy in Federated Learning

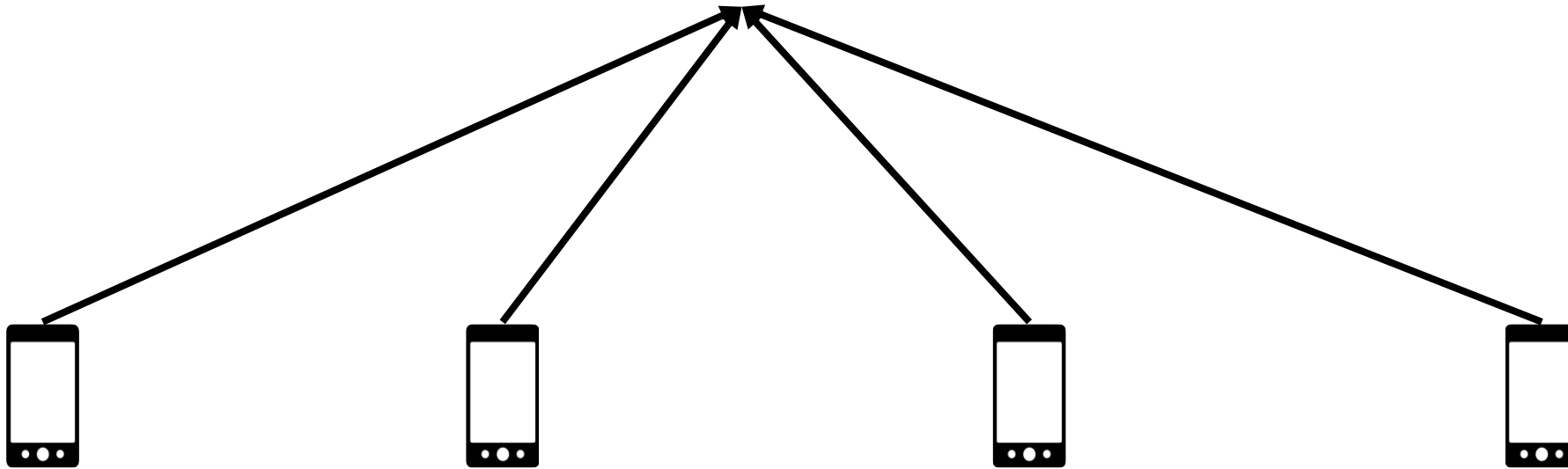
Antonious M. Girgis, Deepesh Data, Suhas Diggavi,  
Peter Kairouz, and Ananda Theertha Suresh

# Context and Motivation



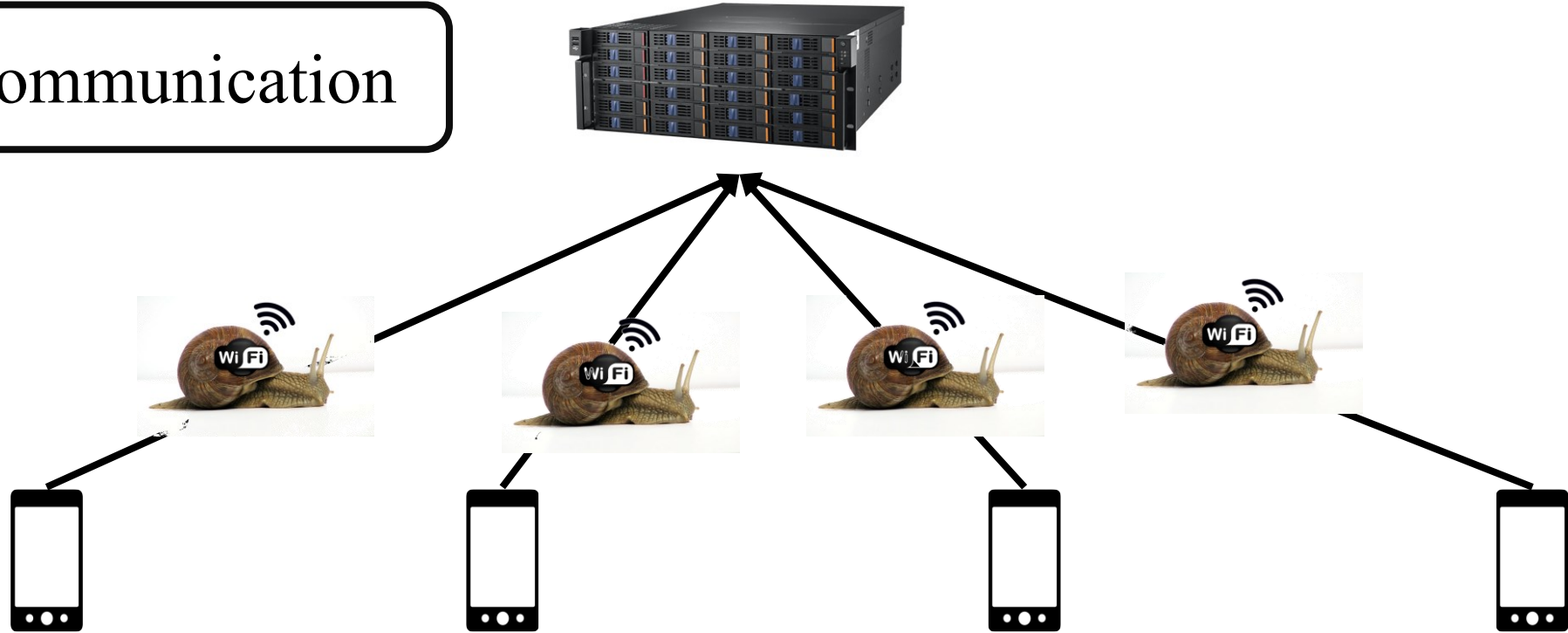
# Context and Motivation

1- Privacy



# Context and Motivation

2- Communication



# Context and Motivation

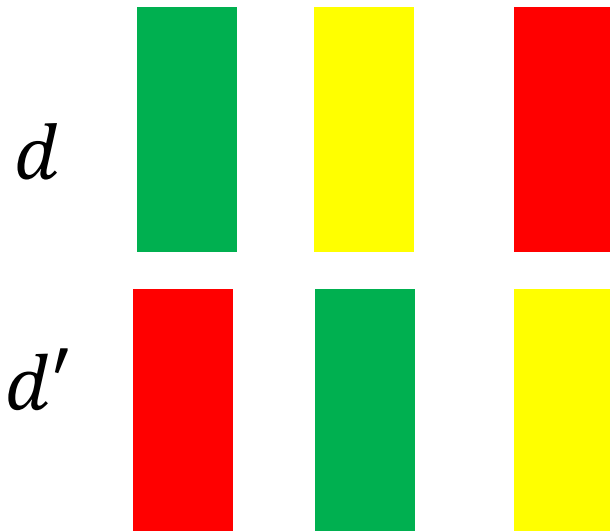


Communication-Efficient and Privacy Preserving Machine  
Learning with good learning performance

# Local Differential Privacy<sup>[Kasiviswanathan et al. 2011]</sup>

Let  $\mathcal{X}$  be a set of all possible inputs

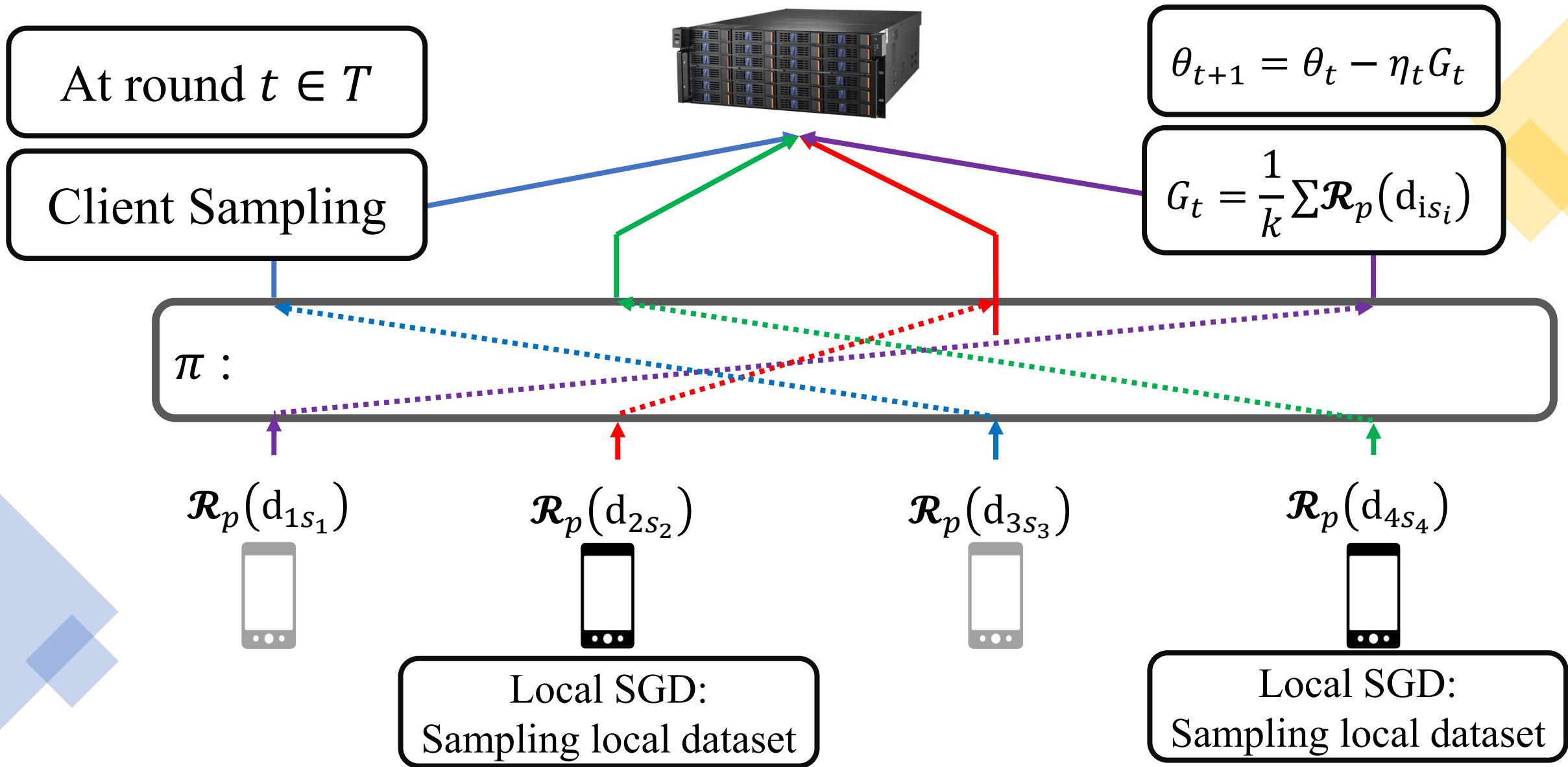
A mechanism  $R$  is  $\epsilon_l$  -  $LDP$  if  
$$\Pr[R(d) \in S] \leq e^{\epsilon_l} \Pr[R(d') \in S] \quad \text{For all } d, d' \in \mathcal{X}$$



**Issue:** Too much randomization/noise is needed.

We need ways to improve this.

# CLDP-Stochastic Gradient Descent (CLDP-SGD)



# Stochastic Gradient Descent (SGD)

Our contributions are mainly two ingredients:

1. Design (unbiased) compressed and private mean estimation under LDP for different  $\ell_p$  geometries that requires  $\mathcal{O}(\log(d))$  bits per gradient.
2. New privacy amplification theorems that combine sampling and shuffling.

**Client/data sampling is non-uniform sampling of data points.**



# Privacy Results of CLDP-SGD

**Theorem.** Choose  $k$  out of  $m$  client at each round

**Privacy:** for  $n = mr$ ,  $q = \frac{k}{n}$ , after  $T$  iterations and  $\epsilon_l = \mathcal{O}(1)$ :

$$\epsilon = \mathcal{O} \left( q \epsilon_l \sqrt{\frac{T \log(1/\delta) \log(T/\delta)}{qn}} \right)$$

- Privacy Amplification by sub-sampling:  $\mathcal{O}(q)$ 
  - Client/data sampling together result in non-uniform sampling from the entire dataset. Existing results cannot be applied.
- Privacy Amplification by Shuffling:  $\mathcal{O} \left( \sqrt{\frac{\log(T/\delta)}{qn}} \right)$
- Strong Composition:  $\mathcal{O} \left( \sqrt{T \log(1/\delta)} \right)$

# CLDP- Stochastic Gradient Descent (CLDP-SGD)

## Theorem.

**Convergence:** For convex, Lipschitz continuous objective  $F$  w.r.t. the dual of  $\ell_p$  -norm, i.e., gradients have bounded  $\ell_p$  -norm for  $p \geq 1$ , CLDP-SGD converges with rate:

$$\mathbb{E}[F(\theta_T)] - F(\theta^*) = \mathcal{O}\left(\frac{D \log T \sqrt{\max(d, d^{2-2/p})}}{\sqrt{qTn} \epsilon_l}\right)$$

**Communication:** Each client sends  $\frac{k}{m} \times \log(2^b)$  bits in expectation per iteration, where  $b = \log(d) + 1$  if  $p \in [1, \infty]$  and  $b = d$  otherwise.

# Main Results:

For  $\ell_2$  Lipschitz functions:

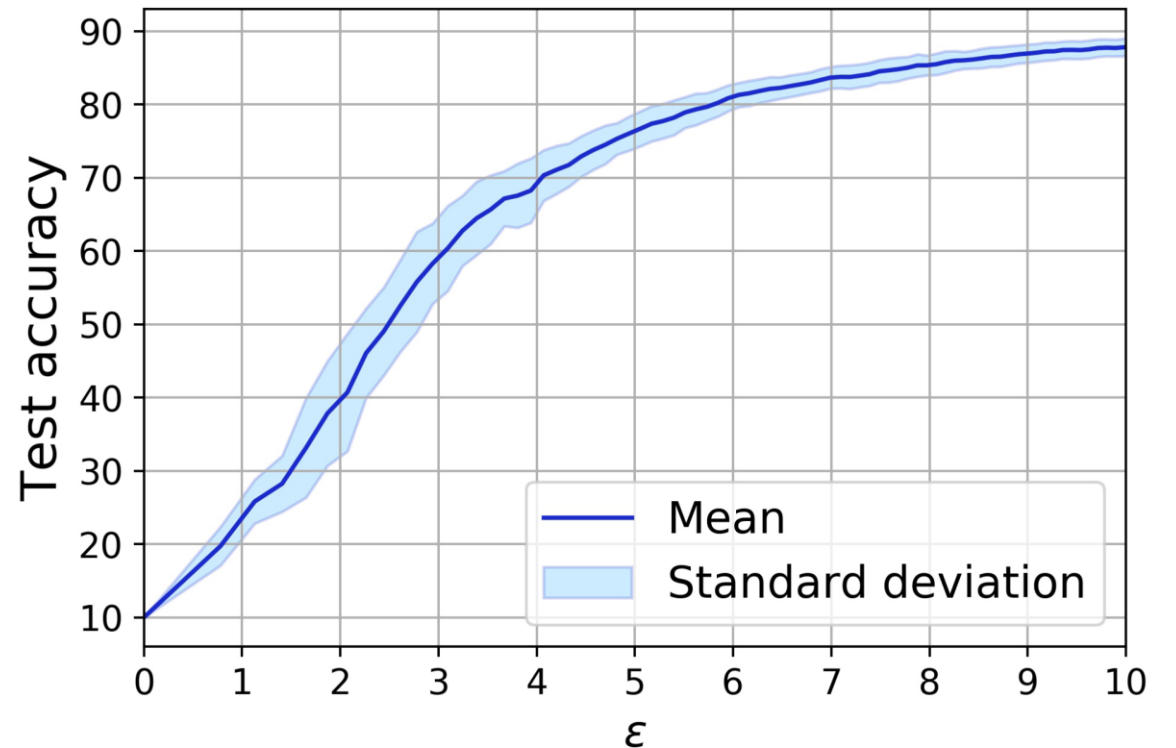
CLDP-SGD algorithm has optimal privacy-convergence rate (same as Bassily et. al [1]) and optimal privacy (same as ESA++[2]), while it uses only  $\log(d) + 1$  bits of communication per sample.

[1] Bassily, Raef, Adam Smith, and Abhradeep Thakurta. "Private empirical risk minimization: Efficient algorithms and tight error bounds." *2014 IEEE 55th Annual Symposium on Foundations of Computer Science*. IEEE, 2014.

[2] U´. Erlingsson, V. Feldman, I. Mironov, A. Raghunathan, S. Song, K. Talwar, and A. Thakurta, "Encode, shuffle, analyze privacy revisited: formalizations and empirical evaluation," arXiv preprint arXiv:2001.03618, 2020.

# Numerical Results:

- MNIST dataset.
- LDP:  $\epsilon_l = 2$
- Privacy loss:  $\delta = 10^{-5}$
- Num. of parameters:  $d = 13,170$
- Communication:  $\log(d) + 1 = 15$  bits per gradient.





**THANK YOU!**