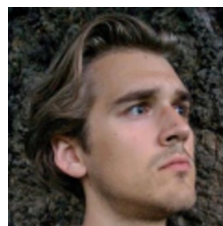


Learning contact dynamics using physically structured neural networks

Andreas Hochlehnert,¹



Alexander Terenin,² Steindór Sæmundsson,² Marc Deisenroth¹

Existing Models:

- Hamiltonian Neural Networks (HNN) ^[1]
Learn Hamiltonian from data
- Variational Integrator Networks (VIN) ^[2]
Variational Integration Scheme



Problem:

No contact dynamics

Aim:

**Introduce a model that can
predict contacts**

[1] Greydanus, Samuel, Misko Dzamba, and Jason Yosinski., Hamiltonian neural networks. *Advances in Neural Information Processing Systems*. 2019.

[2] Saemundsson, Steindor, et al., Variational Integrator Networks for Physically Structured Embeddings. *International Conference on Artificial Intelligence and Statistics*. 2020.

We use **Central-Difference Lagrange**^[1] (CD-Lagrange)

Symplectic Integrator, Capable of simulating
contact, Explicit

[1] Fekak, Fatima-Ezzahra, et al., A new heterogeneous asynchronous explicit-implicit time integrator for nonsmooth dynamics. Computational Mechanics 60.1 (2017): 1-21.

CD-Lagrange Network

$$\mathbf{Q}_{n+1} = \mathbf{Q}_n + h\dot{\mathbf{Q}}_{n+\frac{1}{2}}$$

$$\mathbf{L}_{n+1} = \mathbf{L}(\mathbf{Q}_{n+1}), \quad \hat{\mathbf{c}}_{n+1} = \hat{\mathbf{c}}_{\theta}(\mathbf{Q}_{n+1}, \dot{\mathbf{Q}}_{n+\frac{1}{2}}), \quad \mathbf{H}_{n+1} = \mathbf{H}(\mathbf{Q}_{n+1}, \dot{\mathbf{Q}}_{n+\frac{1}{2}})$$

$$\mathbf{F}_{n+1} = \frac{\partial V(\mathbf{Q}_{n+1})}{\partial \mathbf{Q}}$$

$$\mathbf{v}_{n+\frac{3}{2}} = -e\dot{\mathbf{Q}}_{n+\frac{1}{2}} \mathbf{L}_{n+1}^T,$$

$$\begin{cases} \text{if } \hat{c}_{n+1}^k = 1 & \mathbf{I}_{n+\frac{3}{2}}^k = \mathbf{L}_{n+1}^k \left[\mathbf{H} \left(\mathbf{v}_{n+\frac{3}{2}} + \left(\dot{\mathbf{Q}}_{n+\frac{1}{2}} + h\mathbf{M}^{-1}\mathbf{F}_{n+1} \right) \mathbf{L}_{n+1}^T \right) \right]_{kk} \\ \text{otherwise} & \mathbf{I}_{n+\frac{3}{2}}^k = \mathbf{0} \end{cases}$$

$$\dot{\mathbf{Q}}_{n+\frac{3}{2}} = \dot{\mathbf{Q}}_{n+\frac{1}{2}} + \mathbf{M}^{-1} (h\mathbf{F}_{n+1} + \mathbf{I}_{n+1})$$

V - Potential, $\hat{\mathbf{c}}$ - Contact function, $\mathbf{Q}$ - generalised coordinates

CD-Lagrange Network

Approximated using NN

$$\mathbf{Q}_{n+1} = \mathbf{Q}_n + h\dot{\mathbf{Q}}_{n+\frac{1}{2}}$$

$$\mathbf{L}_{n+1} = \mathbf{L}(\mathbf{Q}_{n+1}), \quad \hat{\mathbf{c}}_{n+1} = \hat{\mathbf{c}}_{\theta}(\mathbf{Q}_{n+1}, \dot{\mathbf{Q}}_{n+\frac{1}{2}}), \quad \mathbf{H}_{n+1} = \mathbf{H}(\mathbf{Q}_{n+1}, \dot{\mathbf{Q}}_{n+\frac{1}{2}})$$

$$\mathbf{F}_{n+1} = \frac{\partial V(\mathbf{Q}_{n+1})}{\partial \mathbf{Q}}$$

$$\mathbf{v}_{n+\frac{3}{2}} = -e\dot{\mathbf{Q}}_{n+\frac{1}{2}}\mathbf{L}_{n+1}^T,$$

$$\begin{cases} \text{if } \hat{c}_{n+1}^k = 1 & \mathbf{I}_{n+\frac{3}{2}}^k = \mathbf{L}_{n+1}^k \left[\mathbf{H} \left(\mathbf{v}_{n+\frac{3}{2}} + \left(\dot{\mathbf{Q}}_{n+\frac{1}{2}} + h\mathbf{M}^{-1}\mathbf{F}_{n+1} \right) \mathbf{L}_{n+1}^T \right) \right]_{kk} \\ \text{otherwise} & \mathbf{I}_{n+\frac{3}{2}}^k = \mathbf{0} \end{cases}$$

$$\dot{\mathbf{Q}}_{n+\frac{3}{2}} = \dot{\mathbf{Q}}_{n+\frac{1}{2}} + \mathbf{M}^{-1} (h\mathbf{F}_{n+1} + \mathbf{I}_{n+1})$$

V - Potential, $\hat{\mathbf{c}}$ - Contact function, $\mathbf{Q}$ - generalised coordinates

Idealised Touch Feedback

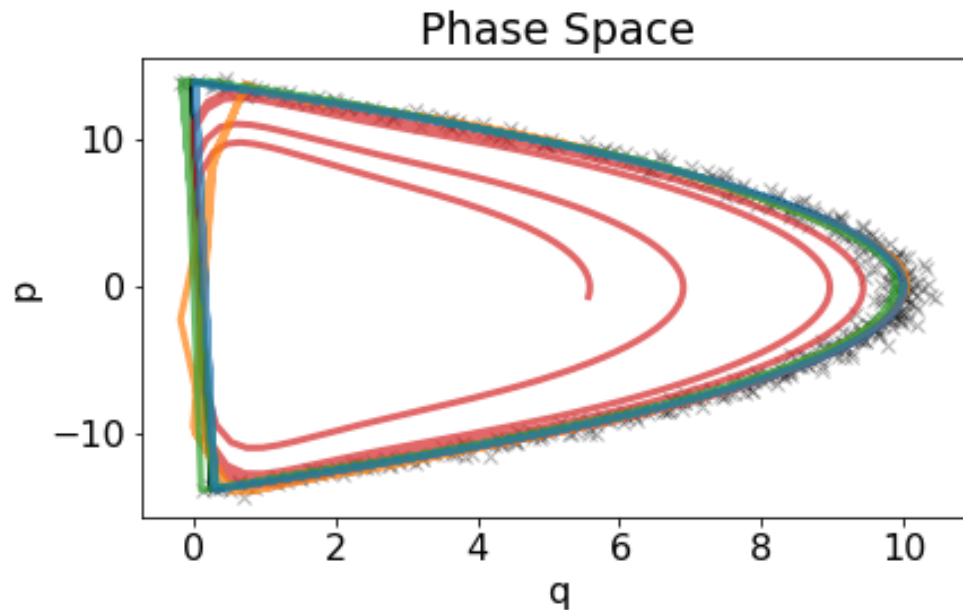
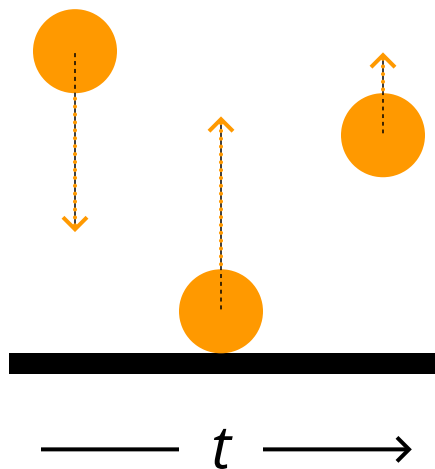
Contact signal

$$c_n^k = \begin{cases} 1 & \text{if contact for body } k \\ 0 & \text{otherwise} \end{cases}$$

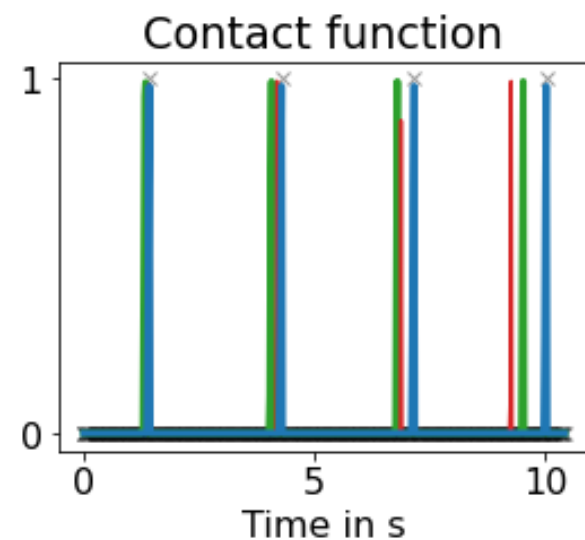
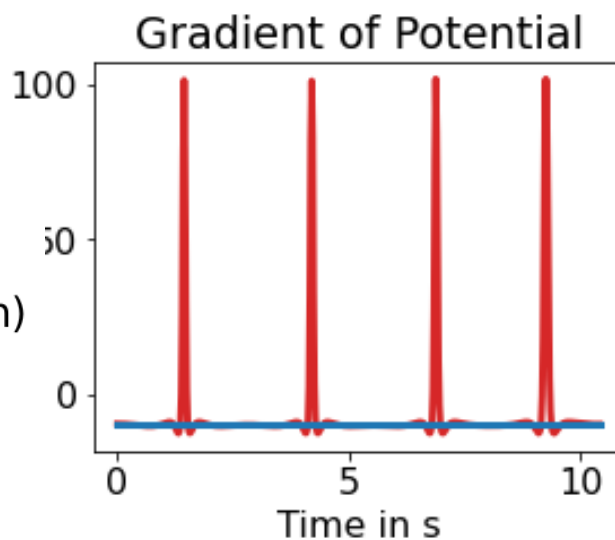
Contact Network

$$\begin{cases} \text{if } \hat{c}_{n+1}^k = 1 & \mathbf{I}_{n+\frac{3}{2}}^k = \mathbf{L}_{n+1}^k \left[\mathbf{H} \left(\mathbf{v}_{n+\frac{3}{2}} + \left(\dot{\mathbf{Q}}_{n+\frac{1}{2}} + h\mathbf{M}^{-1}\mathbf{F}_{n+1} \right) \mathbf{L}_{n+1}^T \right) \right]_{kk} \\ \text{otherwise} & \mathbf{I}_{n+\frac{3}{2}}^k = 0 \end{cases}$$


Bouncing Ball

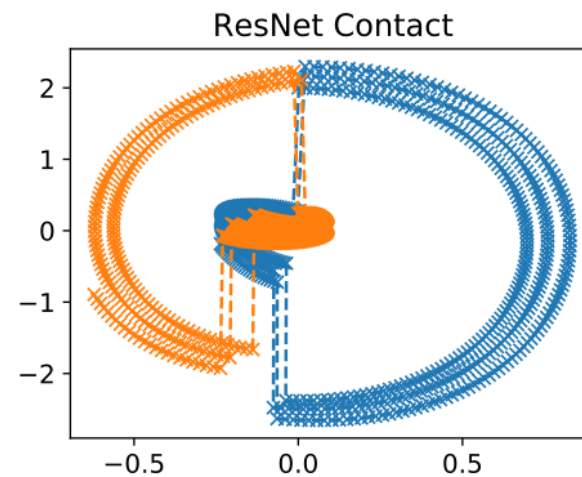
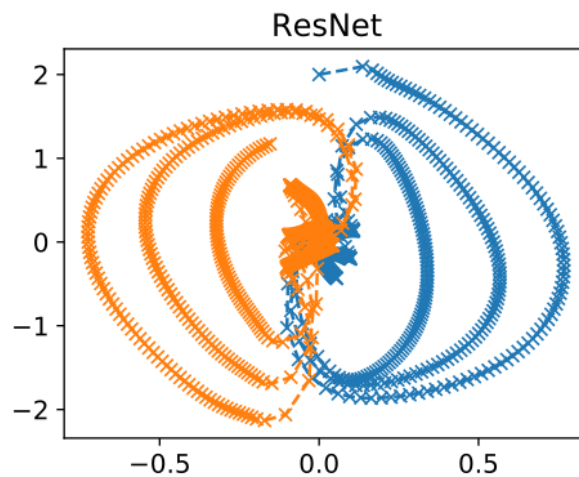
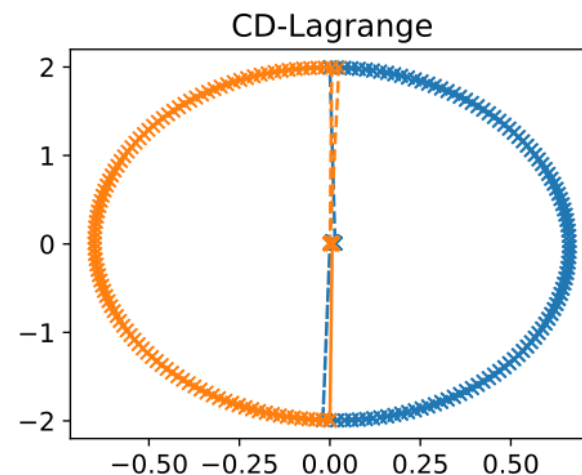
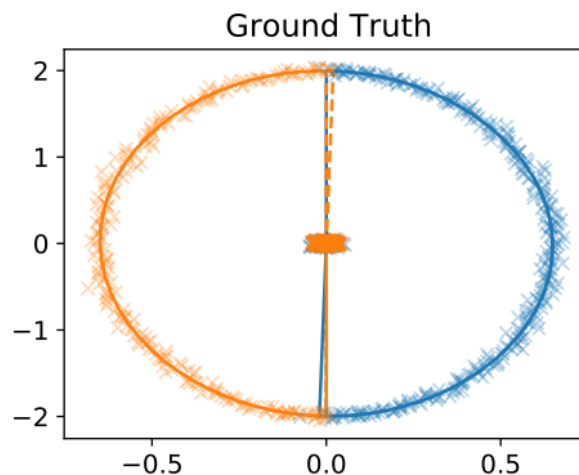
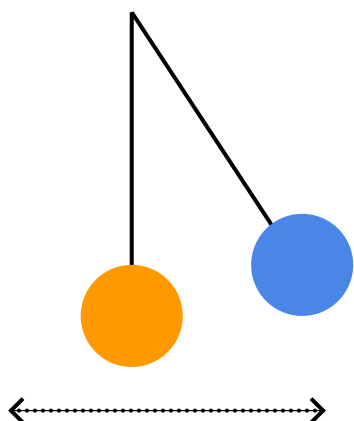


- CD-Lagrange
- ResNet
- ResNetContact
- CD-Lagrange (no touch)
- - - Ground truth
- × Data

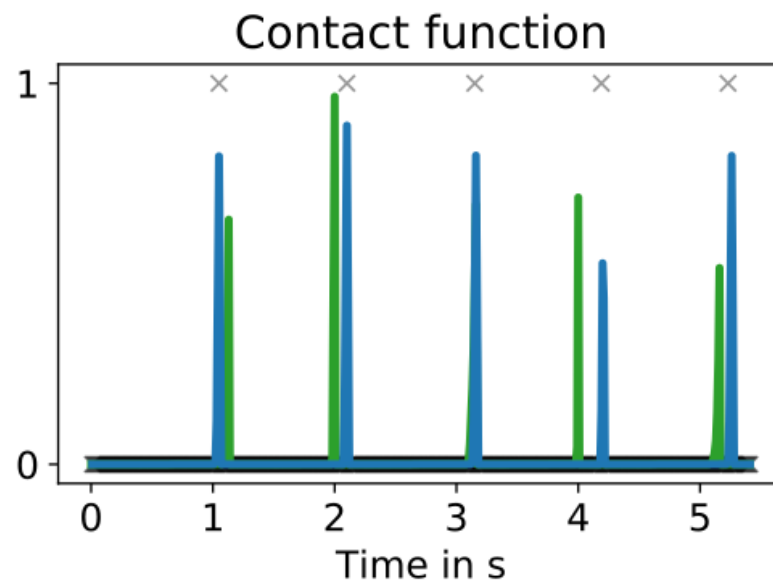
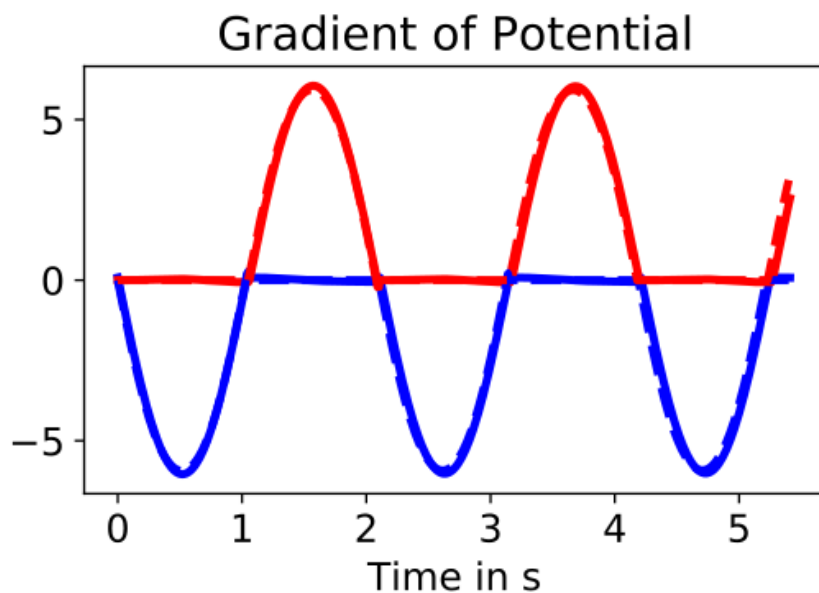


Newton's Cradle

-x- Ball 1
 -x- Ball 2



Newton Cradle



— CD-Lagrange, 1	--- Ground truth, 1	— CD-Lagrange	× Data
— CD-Lagrange, 2	--- Ground truth, 2	— ResNetContact	